d. $\int \frac{2+4\tan(x)}{1+\tan(x)}dx$

$2\int \frac{\frac{1+2\,\text{sen}x}{\cos x}}{1+\frac{\text{sen}x}{\cos x}} = 2\int \frac{\frac{\cos x+2\,\text{sen}x}{\cos x}}{\frac{\cos x+\text{sen}x}{\cos x}} = 2\int \frac{\cos x+2\,\text{sen}x}{\cos x+\text{sen}x}$

$= 2\int \frac{\cos x}{\cos x+\text{sen}x}dx + 4\int \frac{\text{sen}x}{\cos x+\text{sen}x}$

$t = \tan(x/2)$, $\text{sen}x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$, $dx = \frac{2dt}{1+t^2}$

$2\int \frac{\frac{1-t^2}{1+t^2}}{\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2}}\cdot\frac{2dt}{1+t^2} + 4\int \frac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2}}\frac{2dt}{1+t^2}$

$2\int \frac{\frac{1-t^2}{1+t^2}}{\frac{1-t^2+2t}{1+t^2}}\cdot\frac{2dt}{1+t^2}$

$4\int \frac{1-t^2}{(1+t^2)(1+2t-t^2)}$

$1-t^2 = \frac{A(2t)+B}{1+t^2} + \frac{C(2-2t)+D}{(1+2t-t^2)}$

$= (2At+B)(1+2t-t^2) + (2C-2Ct+D)(1+t^2)$

$1-t^2 = 2At+B+4At^2+2Bt-2At^3-Bt^2+2C-2Ct+D+2Ct^2-2Ct^3+Dt^2$

$1-t^2 = (-2A-2C)t^3+(4A-B+2C+D)t^2+(2A+2B-2C)t+(B+2C+D)$

$-2A-2C=0$ | $4A-B+2C+D=-1$ | $2A+2B-2C=0$ | $B+2C+D=1$
$-2A=2C$ | $-4C-2C+2C+1-4C=-1$ | $A+B-C=0$ | $2C+2C+D=1$
$-A=C$ | $-8C=-2$ | $B-2C=0$ | $4C+D=1$
$A=-\frac{1}{4}$ | $C=\frac{1}{4}$ | $B=2C$ | $D=1-4C$
| | $B=\frac{1}{2}$ | $D=1-1$
| | | $D=0$

$4\int \frac{-\frac{1}{4}(2t)+\frac{1}{2}}{1+t^2} + \frac{\frac{1}{4}(2-2t)}{1+2t-t^2}dt$

$4\int \frac{-\frac{1}{4}(2t)}{1+t^2}dt + 4\int\frac{\frac{1}{2}}{1+t^2}dt + 1\int\frac{2-2t}{1+2t-t^2}dt$

$-\ln(1+t^2) + 2\arctan t + \ln(1+2t-t^2)$ 1era parte

② parte

$4\int \frac{2t}{1+2t-t^2}\cdot\frac{2dt}{1+t^2} = 16\int \frac{t}{(1+2t-t^2)(1+t^2)}dt$

$t = \frac{A(2-2t)+B}{1+2t-t^2} + \frac{C(2t)+D}{1+t^2}$

$t = (2A-2At+B)(1+t^2) + (2Ct+D)(1+2t-t^2)$

$t = 2A-2At+B+2At^2-2At^3+Bt^2+2Ct+D+4Ct^2+2Dt-2Ct^3-Dt^2$