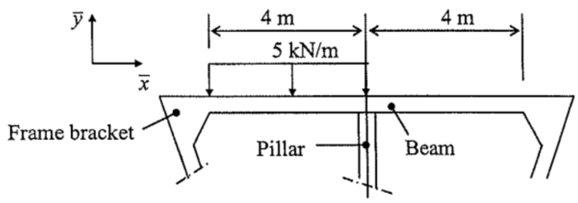
Faculty of Maritime Engineering and Biological, Ocean and Natural Resources Sciences

FINITE ELEMENTS

PARTIAL EXAM 28 June 2016

Student: ....

1. A deck beam of flexural rigidity EI = 6912 kN.m2 subjected to a uniformly distributed load as shown in Figure is supported by frame brackets and a pillar **[40 Pts]**.



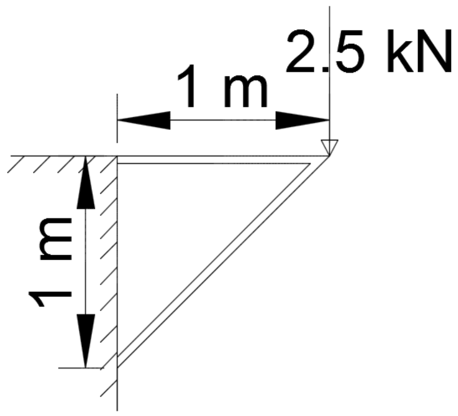
a) Assemble the global structural stiffness matrix of the deck beam using the stiffness matrix of a beam element with length L given below, **[13]**

b) Determine the angular displacement of the beam at the pillar if a uniformly distributed load of 5 kN/m is applied over a span of 4 m. Assume that the pillar constrains only the vertical displacement and the brackets rigidly restrict any movement of the beam ends. **[6]**

c) Calculate all reactions at the brackets and pillar and demonstrate that the beam structure is in equilibrium. **[15]** *Keep in mind the reactions where the external force is applied!*

d) If the brackets were only partially constrained describe how you would solve the beam bracket problem. **[6]**

2. A frame structure attached to the stern of a boat is designed to withstand a vertical load of 2.5 kN as shown below. The cross section of the frame area A of 0.005 m^2 and the second moment of area I of 0.2 x 10^{-4} m^4. Young's Modulus E = 2.1 x 10^8 kN/m^2. **[60 Pts]**



a) Using the stiffness matrix of a general beam element given below, assemble the global structural stiffness matrix of the frame structure.**[11 Marks]**

b) If the frame is rigidly fixed to the stern of the boat, calculate the displacements of the tip of the frame due to the vertical load. **[26 Marks]**

c) Finally, determine all reactions at the stern supports and demonstrate that the frame structure is in equilibrium. **[23 marks]**

<u>Stiffness Matrices:</u>

$$\begin{array}{c} \\ [K^e]=\dfrac{EI}{L^3} \end{array}\begin{array}{c} \begin{array}{cccc} v_i & \theta_{zi} & v_j & \theta_{zj} \end{array} \\ \left[\begin{array}{cc:cc} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ \hdashline -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{array}\right] \end{array}$$

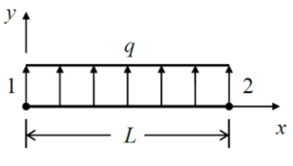
$$[\boldsymbol{K^e}] = \frac{\boldsymbol{EA}}{\boldsymbol{L}}\begin{bmatrix} \boldsymbol{1} & \boldsymbol{-1} \\ \boldsymbol{-1} & \boldsymbol{1} \end{bmatrix}$$

$$[\mathbf{K^e}] = \frac{\boldsymbol{EA}}{\boldsymbol{L}}\begin{bmatrix} \boldsymbol{C^2} & \boldsymbol{CS} & \boldsymbol{-C^2} & \boldsymbol{-CS} \\ \boldsymbol{CS} & \boldsymbol{S^2} & \boldsymbol{-CS} & \boldsymbol{-S^2} \\ \boldsymbol{-C^2} & \boldsymbol{-CS} & \boldsymbol{C^2} & \boldsymbol{CS} \\ \boldsymbol{-CS} & \boldsymbol{-S^2} & \boldsymbol{CS} & \boldsymbol{S^2} \end{bmatrix}$$

$$[\overline{K}^e]=\frac{EA}{L}\left[\begin{array}{ccc:ccc} \bar{u}_i & \bar{v}_i & \theta_{zi} & \bar{u}_j & \bar{v}_j & \theta_{zj} \\ c^2+s^2\dfrac{12r^2}{L^2} & cs-cs\dfrac{12r^2}{L^2} & -s\dfrac{6r^2}{L} & -c^2-s^2\dfrac{12r^2}{L^2} & -cs+cs\dfrac{12r^2}{L^2} & -s\dfrac{6r^2}{L} \\ cs-cs\dfrac{12r^2}{L^2} & s^2+c^2\dfrac{12r^2}{L^2} & c\dfrac{6r^2}{L} & -cs+cs\dfrac{12r^2}{L^2} & -s^2-c^2\dfrac{12r^2}{L^2} & c\dfrac{6r^2}{L} \\ -s\dfrac{6r^2}{L} & c\dfrac{6r^2}{L} & 4r^2 & s\dfrac{6r^2}{L} & -c\dfrac{6r^2}{L} & 2r^2 \\ \hdashline -c^2-s^2\dfrac{12r^2}{L^2} & -cs+cs\dfrac{12r^2}{L^2} & s\dfrac{6r^2}{L} & c^2+s^2\dfrac{12r^2}{L^2} & cs-cs\dfrac{12r^2}{L^2} & s\dfrac{6r^2}{L} \\ -cs+cs\dfrac{12r^2}{L^2} & -s^2-c^2\dfrac{12r^2}{L^2} & -c\dfrac{6r^2}{L} & cs-cs\dfrac{12r^2}{L^2} & s^2+c^2\dfrac{12r^2}{L^2} & -c\dfrac{6r^2}{L} \\ -s\dfrac{6r^2}{L} & c\dfrac{6r^2}{L} & 2r^2 & s\dfrac{6r^2}{L} & -c\dfrac{6r^2}{L} & 4r^2 \end{array}\right]$$

Where C = cosα, S = sinα and r = √(I/A), radius of gyration of cross-section.

And the equivalent nodal loads for a uniformly distributed load q over a beam of length L is:



Uniform load distribution

$$\{F^e\}=\begin{Bmatrix} F_{y1} \\ M_{z1} \\ F_{y2} \\ M_{z2} \end{Bmatrix}=\begin{Bmatrix} \frac{1}{2}qL \\ \frac{1}{12}qL^2 \\ \frac{1}{2}qL \\ -\frac{1}{12}qL^2 \end{Bmatrix}$$

Don't waste your time, select your final matrix accordingly and don't forget to put comments on your results!!