Simple Experimental and Numerical studies of Plate Buckling under Shear loads

Marín López, José R.[1]
García Troncoso, Guillermo[2]

ABSTRACT

Basic formulations to estimate critical load for shear buckling of flat plates were reviewed and applied to two simple situations. Then two configurations of I beams were built, with 3 and 5 vertical stiffeners, with the portions of its webs as the panels previously analyzed. A total of four specimens were built, 100 long, 35 in height and 15 cm in flange width; the webs were 1.5 and the flanges, 3.0 mm in thickness, to avoid compressive buckling of the flanges. Buckling of the panels was observed with the classical inclined pattern; in the second specimen of the first configuration, and both specimens of the second configuration, some weld points failed, with high noise, and a reduction in resistance. Registered buckling vertical loads were: 1708 and 1337 for the first, and 1582 and 1337 kg, for the second configuration. Finally, using SAP2000 computer program, critical loads and corresponding mode shapes were calculated using two numeric models, using 6912 and 7488 Shell elements. Applying load in eight central nodes, 1155 and 2317 kg were necessary to reach the first mode shape of each configuration. In the three types of calculations, the buckling results were similar, that is the laterally deformed panel follow the direction of the main diagonal; the analytical with simply supported edges and numerical results for the critical load show good agreement, while the experimental values, from the second configuration are below the expected values.

Key words: Plate shear, Buckling experiment

1. Overview

Buckling is a very important phenomenon which must be considered in structural design of ships; it is recognized as an unstable equilibrium, and a very small perturbation may take the system to a very different position, usually with large amplitude. After it occurs, the structure results with heavy damage, and the element related with high reduction in its resistance. Usually it is associated with compressive load, but it may also be produced by high values of shear stresses.

In ships, this situation may appear when the hull rests on a docking bed, with very high concentrated forces on the vertical plate of the keel, or, when considering the very high weight of a propulsion engine resting on its foundations. A tall structural element (for example a vertical plate forming the web of an I beam) has very large bending resistance, but because of the large internal inplane force developed, it must be checked against shear stress: to avoid yielding and also buckling.

Since it is not very common to develop shear buckling calculations in our region, it was decided to review formulations to estimate the critical stress. But it was felt that experimental observation of the phenomena was needed. Finally, it was considered useful to apply the available software to estimate using the Finite element method, the critical buckling force and corresponding mode shapes. This effort was undertaken as an undergraduate thesis, (García 2011), and summarized in this work.

2. Review of the estimation of Critical load

The classical book by Bleich, (Bleich, 1952) describes a process developed by Timoshenko to estimate the critical buckling load using Minimum Potential Energy principle, for a rectangular flat plate of constant thickness, subject to uniform shear load.

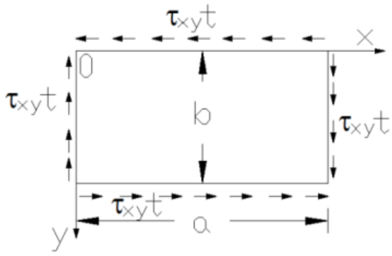
---

[1] Facultad de Ingeniería Marítima y Ciencias Biológicas, Oceánicas y de Recursos Naturales, Escuela Superior Politécnica del Litoral, Profesor, Km. 30.5 Vía Perimetral, Guayaquil, Ecuador. Correo-e: jrmarin@espol.edu.ec

[2] Ingeniero Naval. Correo-e: gxgarcia@espol.edu.ec

Este artículo está siendo revisado para publicarse en una revista técnica.

Figure 1.- Geometry of a plate panel subject to uniform shear load



According to the mentioned principle, the Potential energy reaches an extreme value at equilibrium. The Total potential energy is evaluated as the sum of the Bending internal energy, *U*, and the change in work developed by the external forces, *V*:

$$U + V = constant \tag{1}$$

Additionally, the following symbology will be used:

w: Plate buckling lateral deflection perpendicular to the xy plane,
D: Bending rigidity of the plate, $= Et^3 / [12(1-\nu^2)]$,
E: Young modulus of the material,
ν : Poisson ratio of the material, and
t: Uniform thickness of the plate.

The Bending strain energy is calculated by:

$$V = \frac{D}{2}\int_0^a\int_0^b \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) - 2(1-\upsilon)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} dxdy$$

And specifically for simply supported or clamped edges, the second term of the integral vanishes, leaving:

$$V = \frac{D}{2}\int_0^a\int_0^b \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) dxdy \tag{2}$$

The change in potential energy of the external load may be expressed as:

$$U = -\tau_{xy} t \int_0^a\int_0^b \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} dxdy \tag{3}$$

Using Ritz method, the deflection w is expressed as a combination of functions that satisfy the Boundary conditions of the problem. In the case of a simply supported plate, *w* may expressed as:

$$w = \sum_{i=1}^{n}\sum_{j=1}^{n} f_{ij}\, sen\left(\frac{i\pi x}{a} \right) sen\left(\frac{j\pi y}{b} \right)$$

Replacing in (2), and developing the integrals, the following expression is obtained:

$$V = \frac{\pi^4}{8} Dab \sum_{i=1}^{n}\sum_{j=1}^{n} f^2{}_{ij} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)^2$$

Similarly, replacing the expression for w in the potential energy of the external load, and after integration:

$$U = -4\tau_{xy} t \sum_{i=1}^{n}\sum_{j=1}^{n} f_{ij} f_{i'j'} \frac{iji'j'}{(i^2 - i'^2)(j'^2 - j^2)},$$

where $i+i'$, and $j+j'$, are odd integers.

Denoting the plate aspect ratio as $\alpha = a/b$, and taking the leading terms of the series ($i=j=1$, $i'=j'=2$) the Total potential energy is:

$$V + U = \frac{\pi^4}{8}\frac{D}{\alpha^3 b^2}\left[f_{11}^2 (1+\alpha^2)^2 + f_{12}^2 (1+4\alpha^2)^2 + f_{21}^2 (1+\alpha^2)^2 + f_{22}^2 (1+4\alpha^2)^2 \right] + 8\tau_{xy} t\left(\frac{4}{9} f_{11} f_{22} - \frac{4}{9} f_{12} f_{21}\right)$$

Differentiating with respect to the f_{ij}'s, and equaling to zero, the following system is obtained:

$$\begin{array}{lllll}
B(1+\alpha^2)^2 f_{11} & & & +\frac{4}{9}\tau_{xy} f_{22} & = \mathbf{0} \\
 & B(1+4\alpha^2)^2 f_{12} & -\frac{4}{9}\tau_{xy} f_{21} & & = \mathbf{0} \\
 & -\frac{4}{9}\tau_{xy} f_{12} & +B(1+\alpha^2)^2 f_{21} & & = \mathbf{0} \\
\frac{4}{9}\tau_{xy} f_{11} & & & +B(1+4\alpha^2)^2 f_{22} & = \mathbf{0}
\end{array} \qquad (4)$$

here the constant B represents $B = \frac{\pi^4}{32}\frac{D}{\alpha^3 b^2 t}$.

The above deduced system of equations is homogeneous in the f_{ij}'s, and to obtain a nontrivial solution, its determinant must be set equal to zero, producing a group of values for τ_{xy}:

$$\tau_{1,2} = \pm\frac{9}{4}B(1+\alpha^2)(1+4\alpha^2)$$
$$\tau_{3,4} = \pm\frac{9}{4}B(1+4\alpha^2)(1+\alpha^2)$$

The above results show that the buckling may be caused either by a positive or negative shear. Taking the lowest value, and replacing B, results:

$$\tau_c = \frac{9}{32} * \frac{\pi^4 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 * \frac{(1+\alpha^2)^2}{\alpha^3}$$

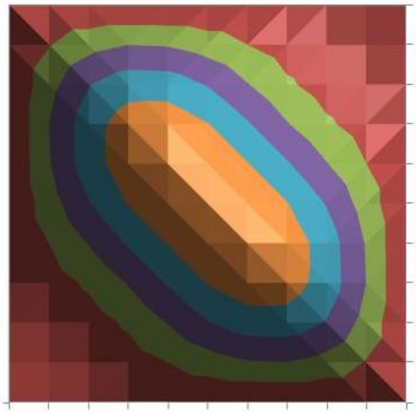
Finally, the above expression in rewritten in a form which is common in plate buckling:

$$\tau_c = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 k$$

where:
$$k = \frac{9\pi^2}{32}\frac{(1+\alpha^2)^2}{\alpha^3}.$$

Since the system of equations (4), is linearly dependent, as an exercise the last three equations were used to obtain the values of the ratios f_{ij}/f_{11}, obtaining the following results: $f_{12}/f_{11}=0.0$, $f_{21}/f_{11}=0.0$, and $f_{22}/f_{11}=-0.2673$, which were combined graphically as shown in the next figure:

Figure 2.- First mode shape



The above procedure includes just a few terms of the expansion, and several authors have contributed with expressions more terms. With those results, Bleich presents the equation for the constant k:

$$k = 5.34 + \frac{4}{\alpha^2}, \quad for\, \alpha > 1.$$

Similarly, for clamped rectangular plates, Bleich presents the following expression to evaluate k:

$$k = 8.98 + \frac{5.60}{\alpha^2}, \quad for\, \alpha > 1.$$

Applying the above equation to two plate panels with a dimensions equal to 45 and 22.5 cm, and, b dimension equal to 35 cm and 1.5 mm in thickness, and, considering that they are built from H14 aluminum alloy[3], the shear critical stress is presented in the following table. Also, multiplying the critical shear stress times the area of the web (*35*0.15* cm^2), the total force on the vertical sides is estimated, and named $F_{cr}/2$; this parameter will be later compared with the vertical force to buckle I beams having the same dimensions for the panels in its webs.

Table 1.- Critical shear stress for a flat plate

	a/b=1.286			*a/b=0.643*		
	k	τ_{cr}, kg/cm^2	$F_{cr}/2$, kg	k	τ_{cr}, kg/cm^2	$F_{cr}/2$, kg
Simply supported edges	*7.76*	*95.24*	*500*	*16.92*	*207.69*	*1090*
Clamped edges	*12.37*	*151.80*	*797*	*27.33*	*312.85*	*1642*

Considering an elastic limit of 616 kg/cm^2, the above results show that the buckling will appear within the elastic limit of the material.

3. Experimental work

Two I beams were built from H14 aluminum alloy, due to its low price and easiness to be acquired locally in plates with small thickness. The beams were built of about 100 cm long, 35 in height, and 15 in breadth. The web was 1.5 and the flanges 3.0 mm in thickness, and the elements were joined with intermittent TIG welding. Two configurations were selected to change the aspect ratio of the panel, named A and B, installing an extra transversal stiffener; the portions of the webs in the I beams correspond to the panels analyzed in the previous section. After a previous experience, (Rodríguez 2009), to avoid local failures extra stiffeners were installed at the ends and at the center of each beam. Finally, due to a shortage of budge only two copies of each configuration were built, as shown in next figure.

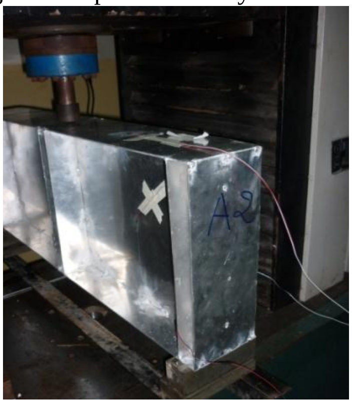
[3] For this alloy, the following properties were considered: E=723998.5 kg/cm^2, and ν=0.33 (//azom.com). Also the tensile elastic limit is 1120 kg/cm^2, and the Shear Yield stress is approximated as 0.55*TEL=616 kg/cm^2.(Taken from //lumetalplastic.com/ and //en. wikipedia.org/)

Figure 3.- Specimens built.

After the specimens were built, predeformations were measured, using a very stiff angle and sliding a dial gage along it, (García, 2011). Joining the first and last measurements with a straight line, the differences with respect to that reference were calculated. The maximum values of the predeformations were: 5.39 for A1, 2.91 for A2, 2.21 for B1, and, 1.22 mm for B2 specimen. When compared with the thickness of the web plate, 1.5 mm, they are found very large, so in general is expected to have a high influence on the critical buckling forces.

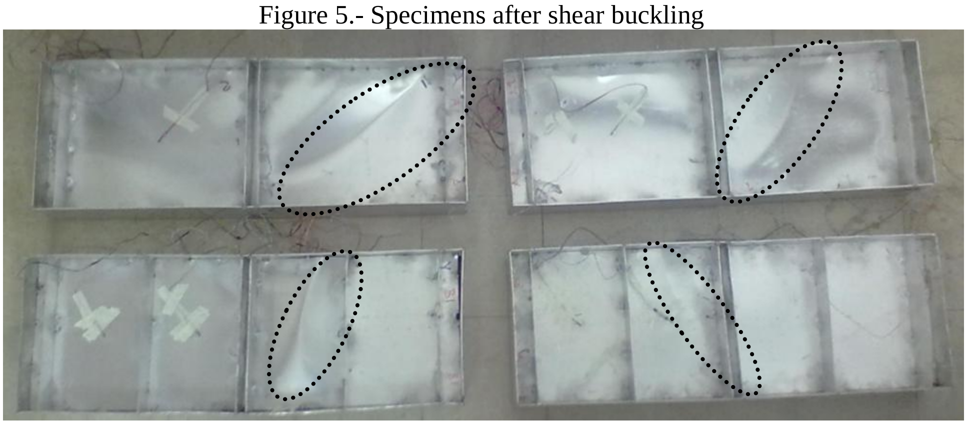
A vertical load was applied in the center point using a 4.8 cm steel cylinder, using an Universal test machine. Also strain gages were installed: on the opposite surfaces of the plate in the directions of the main planes (half bridge), at the center of the panel, to register shear strain, and, longitudinally (quarter bridge) on the flange, to register normal bending strain.

Figure 4.- Specimen ready to be tested



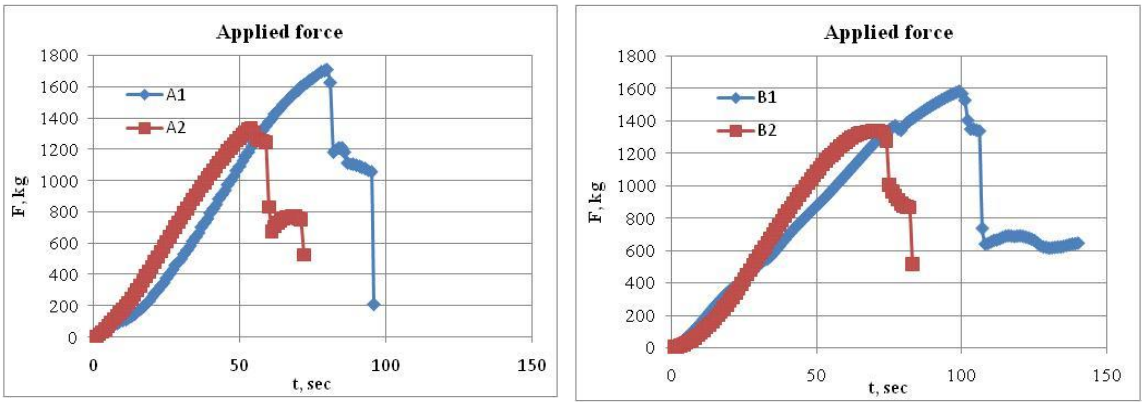
In the following figure, the buckled specimens are shown. The buckled panel of the A1 specimen shows the deformation inclined with the main diagonal. In the A2 panel, the deformation did not follow completely the direction of the main diagonal because of a failure of a weld point.

Figure 5.- Specimens after shear buckling



Following there are shown the variation of the applied forces for each of the configuration.

Figure 6.- Applied force for shear buckling of each specimen



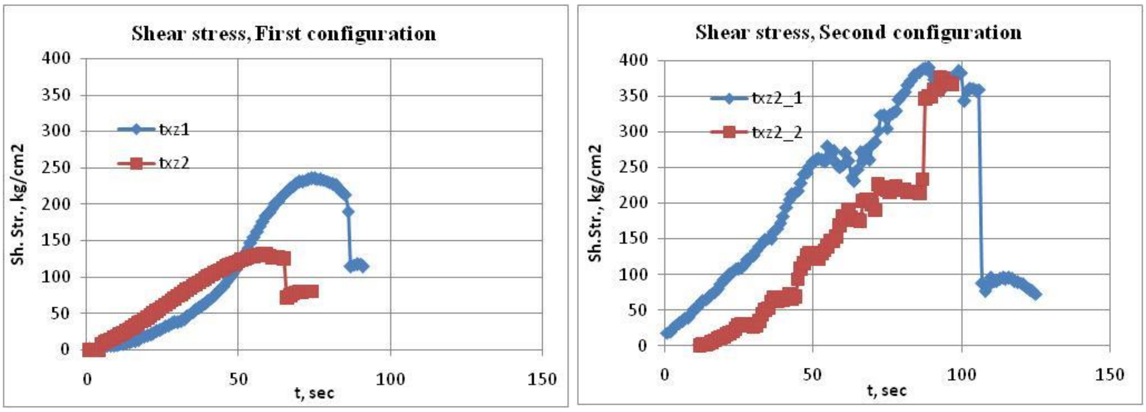
A large reduction in applied force is observed, denoting the buckling of the specimen. In the following table the maximum values of the applied forces are presented. They show a large variation between each other:

Table 2.- Maximum and average applied forces

A1	1707.9 kg	B1	1582 kg
A2	1337 kg	B2	1337.3 kg
A_{aver}	1522.5 kg	B_{aver}	1459.7 kg

With the registered values of the shear strain, at the mid height of each specimens, the corresponding shear stress was calculated. In general as can be seen, all tests were below the Elastic limit of the material:

Figure 7.- Shear stress at center of panel

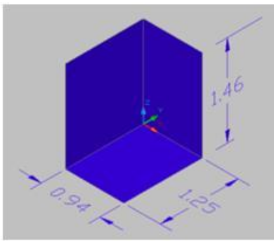


The first specimen of the first configuration, presents a curve with increase in slope, while the second presents a curve with a straight part, and then a soft reduction in slope, until the buckling occurs. In the second configuration, the variation of shear stress shows not exactly a continuous curve, and the second one increases after the buckling. As a comment, since the panels of the second configuration have a smaller a dimension, 22.5 cm, the distortions resulting from the welding produced these strange measurements.

4. Numerical estimation

As a final part of the project, the critical shear stress was calculated using a Finite element computer program, SAP2000, (CSI, 2000), which has the capability to handle nonlinear problems. The model tried to represent closely the two configurations, with 6912 and 7488 SHELL elements, for configuration A and B. Following there are shown the dimensions of the Shell elements of the web and flange:

Figure 8.- Dimensions of the Shell elements used for the discrete representation of the beams



For the calculation, 100 kg of vertical force was applied on each of 8 central nodes, forming a rhombus, and the critical force to produce the first six buckling modes were:

Table 3.- Critical force for the first six modes for configurations A and B

	Critical force [kg]	
Mode #	A	B
1	1155	2317
2	1156	2330
3	1250	2540
4	1253	2545
5	-1975	2609
6	-1978	2610

In the following figures, the deformed pattern, and the spatial distribution of the local bending moments for the first mode shape are presented, for both configurations. It is clearly seen the same inclined deformation in the buckled panel of the webs. Also in configuration B, the internal plate is the one that buckles first.

Figure 9.- Mode 1 for first configuration

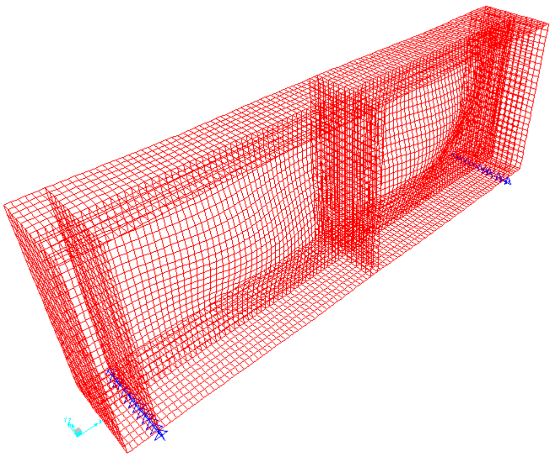


Figure 10.- Distribution of lateral deformation for 1st mode, first configuration

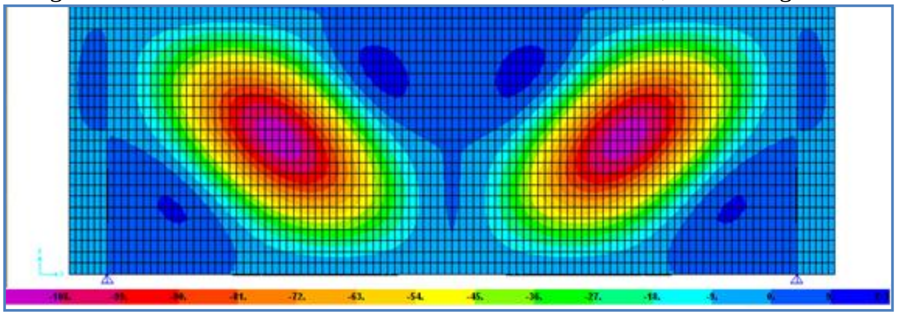


Figure 11.- Distribution of bending moment 11 for 1st mode, first configuration

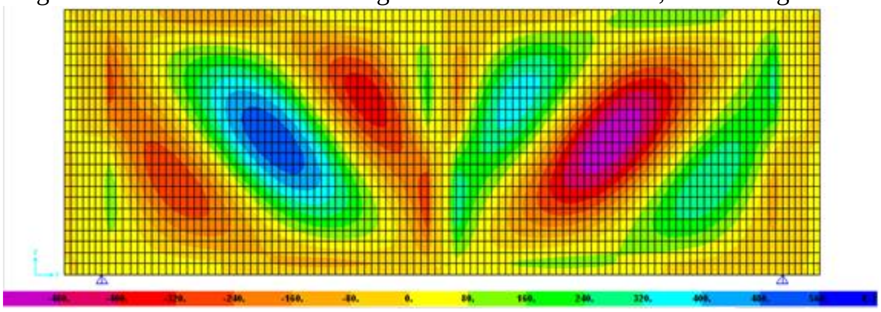


Figure 12.- Distribution of bending moment 22 for 1st mode, first configuration

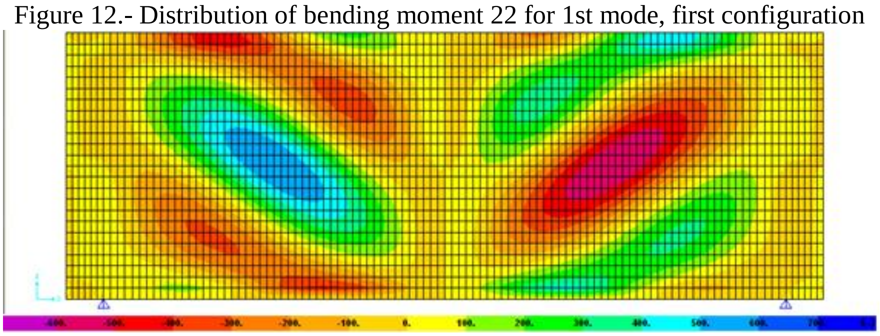


Figure 13.- Mode 1 for second configuration

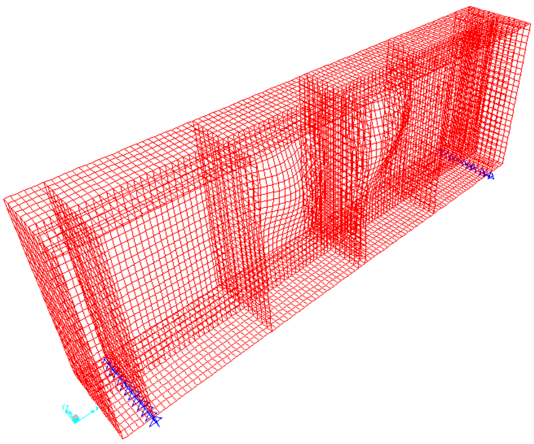


Figure 14.- Distribution of lateral deformation for 1st mode, first configuration

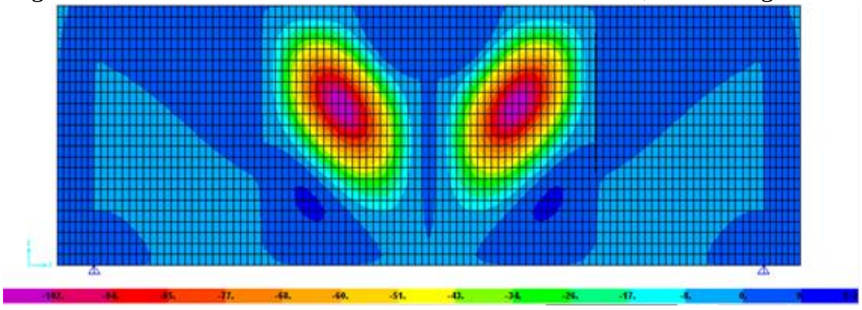


Figure 15.- Distribution of bending moment 11 for 1st mode, second configuration

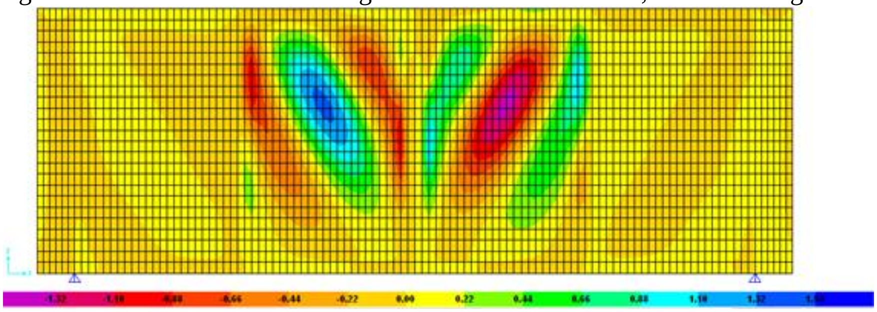
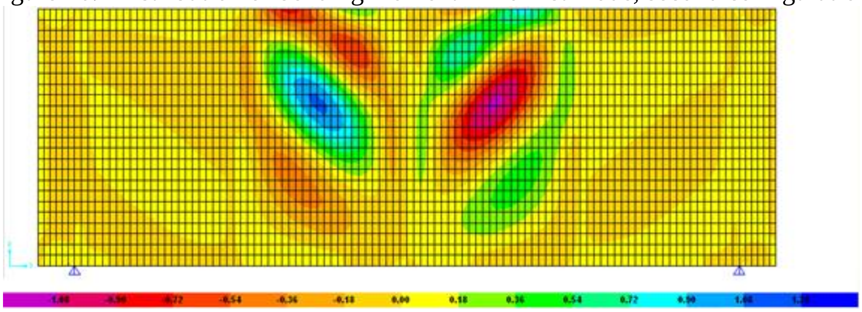


Figure 16.- Distribution of bending moment 22 for 1st mode, second configuration



5.- Final Comments.-

A short investigation was developed to understand the shear buckling phenomena of flat plates. After reviewing some available analytical expressions from a classical reference to estimate critical stress, they were applied to two panels with doubled plate aspect ratio. Then a total of four I beams of two configurations were built, using aluminum alloy H14, Length: 90, Height: 35, and Flange width: 15 cm, with web and flange thicknesses of 1.5 and 3.0 mm, respectively. The beams have 3 and 6 vertical stiffeners, so that the portions of the web correspond to the two panels previously analyzed. The maximum predeformations of the specimens were in the order of 5.39 mm, resulting from the welding of thin plates. Finally, a FEM model was prepared using Shell elements, and the critical loads and corresponding mode shapes were determined.

The results show similar buckled profile, that is, a lateral deflection that follows the main diagonal of the panel. In the case of the analytical results, as expected, the boundary conditions, Simply supported vs clamped, influence heavily on the critical stresses.

Bibliography

BANNERMAN, D., and JAN, H., Analysis and Design of Principal Hull Structure, chapter VI in Ship Design and Construction, Taggart, R., ed.. SNAME, 1980

BLEICH, Buckling Strength of Metal Structures. McGraw-Hill Book Company, 1952

Computers and Structures, Inc., SAP2000 Linear and Nonlinear Static and Dynamic Analysis and Design of Three-Dimensional Structures. CSI, 2010

GARCÍA, G., Estudio del Pandeo de Paneles de Planchaje sometidos principalmente a Carga Cortante. Tesis de Grado de Ing. Naval, Fac. Ingeniería Marítima y Ciencias Biológicas, Oceánicas y de Recursos Naturales, ESPOL, Guayaquil, Ecuador, 2011

RODRÍGUEZ, J., Comprobación Experimental de las Cargas de Pandeo de Planchas con Refuerzos. Tesis de Grado de Ing. Naval, Fac. Ingeniería Marítima y Ciencias del Mar, ESPOL, Guayaquil, Ecuador, 2009