ESCUELA SUPERIOR POLITÉCNICA DEL LITORAL

Facultad de Ingeniería en Electricidad y Computación

"SIMULACIÓN DE SISTEMAS DE CANCELACIÓN CIEGOS EN SISTEMAS DMT."

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A todos los que luchan por un

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DECLARACIÓN EXPRESA

"La responsabilidad del contenido de esta Tesis de Grado, me corresponde exclusivamente; y el patrimonio intelectual de la misma a la Escuela Superior Politécnica del Litoral".

(Reglamento de exámenes y títulos profesionales de la ESPOL).

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RESUMEN

Este trabajo ha sido desarrollado con la finalidad de dar a conocer la mejor forma de realizar una cancelación de diafonía cuando se halla en un escenario de VDSL2, basándose en dos métodos, uno ideal y el otro una aproximación al método ideal, denominado método ciego, mientras revisamos las características en las que se desarrolla esta tecnología lo que veremos es su desarrollo matemático para poder ser realizado. Es muy útil saber de dónde se obtienen aquellas variables que están siendo calculadas dentro de una ecuación que define la cancelación de diafonía y además obtener nuevos conocimientos que nos muestren que no solo hay interferencias externas en los medios de comunicación sino que además hay internamente problemas en el canal de comunicación.

Hoy por hoy estamos en la evolución de la tecnología VDSL, esperemos que los inconvenientes de aplicación sean minimizados con el algoritmo de cancelación ciega ya que es muy eficiente en reducir sus interferencias, teniendo presente que el mejor algoritmo de cancelación es el ideal, pero que lamentablemente no estamos dispuestos a implementar debido a una de sus variables físicas. Pero aquí podremos obtener más que una idea de este algoritmo y aprender su punto de aplicación, cual es su desempeño y en donde está la dificultad de ser aplicado, teniendo presente que hay otros métodos de cancelación de diafonía, entenderemos porque el algoritmo de cancelación ciego es el más accesible, y nuestros compañeros podrán tener presente si esta tecnología les es

conveniente en su debido momento que se necesite realizar estudios de aplicación de VDSL2.

Como estudiantes de Electrónica y Telecomunicaciones estamos capacitados para realizar un análisis a esta tecnología, haciendo uso de conceptos básicos y de la experiencia obtenida a lo largo de la carrera, pudiendo evaluar características y datos que en un momento estudiantes nos cuestionamos de su importancia, por eso, mediante el presente trabajo, se podrá ver una total aplicación de aquellos conceptos de señales y sistemas, el valor que se juega la estadística y sobre todo la probabilidad, inmersas en ecuaciones que se apegan potencialmente a espacios reales. Con esto les hacemos un llamado a todos los compañeros de la carrera, para que no lleguen a pensar que el material que se revisa no es necesario, sino más bien es tan útil como importante para involucrarse en las nuevas tecnologías que están inundando el medio actual.

Queremos dar paso a mas investigaciones sobre cancelación de diafonía, a pesar de existir métodos actuales, en la práctica las variables costos siempre pone un alto en toda aplicación, por esto queremos ser parte de la iniciativa hacia la investigación en VDSL2, ya que hay además mucho de donde investigar, por ejemplo el retardo entre símbolos, la aproximación en el exceso de banda y mas temas que hemos presentado en nuestro trabajo..

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INTRODUCCIÓN

En la actualidad disponemos de una tecnología que permite compartir conocimientos de manera ilimitada del mundo como es el internet, casi todas las personas que poseen un computador están conectados a internet, pero conseguir este servicio tiene sus costos y siempre buscamos economizar al momento de adquirirlo, una de las maneras de economizar el costo del servicio por parte de los proveedores es haciendo uso de redes que ya están en todas partes y no exista la necesidad de crear una nueva red, hablamos de las redes públicas conmutadas de telefonía análoga para voz, la cual está ya presente y que nos ayuda a reducir costos de implementación de redes de datos. Mantener esta red en buenas condiciones solo nos presenta pequeños costos operativos que a lo largo de los años se debe realizar para mantener calidad de servicio. Esta tecnología que mencionamos se la llama DSL, Línea Digital de Abonado (Digital Subscriber Line), y está en constante desarrollo debido a las limitantes de conexión y velocidad para la transmisión de datos, sin mencionar la topología de la red, esta tecnología puede llegar a una velocidad de transmisión de 100 Megabits por segundo tanto en descarga como en subida (VDSL2).

El sentido de hacer uso de esta tecnología es dar al usuario varios servicios bajo una topología de red, servicio conocido como "triple play", que incluye voz, video, datos, televisión de alta definición y juegos interactivos, pero la falta de base tecnológica ha postergado esa posibilidad de crecimiento, a pesar de la gran demanda por parte de los usuarios de una tecnología que optimice su tiempo brindando mayor velocidad de respuesta y la obtención de gran cantidad de información como es el caso de VDSL2, la que aprovecha más apropiadamente el espectro de frecuencia del cobre, utilizando cuatro canales para la comunicación, dos para la subida (del cliente hacia el proveedor) y dos para la bajada.

El problema de las tecnologías en vías de desarrollo es que necesitan una regulación y en la práctica necesitan cumplir factores que no la expongan a efectos no deseados como interferencia, atenuación de la señal por distancia del cliente, retardos en la comunicación, entre otros factores, es así, que la evolución de estas tecnologías trae consigo el desarrollo de equipos que corrijan errores en la transmisión; están los repetidores de señal, aislantes de ruido externo, pero aun asi estos factores externos no son lo suficiente para mantener una señal limpia que transporte los datos claramente, pues, existen también factores internos que se producen en el canal de comunicación, como el eco, la diafonía, desvanecimiento, etc., pero gran parte de ellos son corregibles mediante equipos o software inteligente. Nuestro mayor problema está en la diafonía, causada por acoplamientos magnéticos entre los pares de cables o como consecuencia de desequilibrios de admitancia entre los mismos hilos que forman el circuito de comunicación, difíciles de corregir ya que no existe un común denominador entre los fabricantes de cables, revestimiento de estos y demás características que hacen al cable realizar un enlace. Los problemas que nos trae la diafonía son bien serios en la comunicación ya que hace que se vea limitado el rendimiento del sistema VDSL2, afectando parámetros como la capacidad del canal y tasa de error, además de crear la interferencia entre un cable y otro, teniendo en cuenta que existen cables con más de un par trenzado en su interior nos limita a aumentar el número de abonados por un mismo cable y además no podemos ponerlos bien cercanos entre sí, porque se crea una distorsión de información que no podrá ser procesada luego por nuestro modem de comunicación, quedando esta tecnología descartada para su uso.

El despliegue de esta tecnología hace que el siguiente trabajo tenga una relevante importancia en su desarrollo ya que vamos a realizar una eliminación o cancelación de diafonía, tratar de limpiar este problema que se presenta entre los cables de varios pares trenzados en su interior y así traer la tecnología a nuestro país. Somos conscientes de que todos los usuarios de internet necesitan de una conexión lo mas económica posible y con una fuerte eficiencia en la comunicación de sus datos, es por eso que creemos muy importante el desarrollo de este tema y esperamos que sea tomado presente para nuevas tecnologías como son del grupo xDSL.

CAPÍTULO I.

1 CONTENIDO

1.1 Técnicas de cancelación

Existen varios criterios para poder realizar una cancelación de diafonía en un canal VDSL2, los cuales se pueden emplear del lado del transmisor o del lado del receptor.

Del lado del transmisor hay varias técnicas como:

El uso de códigos de bloqueo: Si los bloques de código son mayores que la longitud mínima, el NEXT puede eliminarse por completo;

Modificando la configuración de los espectros de transmisión: diseñado para rechazar la NEXT de una manera que maximiza la tasa general de los datos, además mantiene la compatibilidad espectral con otros servicios;

Modificando el algoritmo de carga de bit: cambia el orden de poder colocar los bits en la trama.

Pero estas técnicas mencionadas están solo enfocadas a evitar la diafonía, mas no realizan la cancelación total de esta, por otro lado, su implementación prácticamente tiene una excesiva dificultad, ya que requieren de que se realicen cambios en los estándares existentes para la comunicación VDSL2. De esta manera no es la mejor opción para nosotros tomar una de estas técnicas de cancelación.

Las técnicas que disponemos por el lado del receptor son:

La ecualización por retroalimentación: la diafonía asume tiene el mismo índice de muestreo como fuente, útiles para cancelar su propio NEXT y su propio FEXT;

La detección de multiusuario y sus variaciones ciegas posiblemente sean las mejores técnicas para hacer la cancelación de diafonía, sin embargo, al ser tan generales hacen que su desarrollo computacional sea tan costoso de implementar.

La mayoría de técnicas para cancelar o atenuar la diafonía requieren el conocimiento explicito de la función de acoplamiento que se presenta entre los pares trenzados, la cual no es posible obtener debido a que no hay una característica exacta de estos entre la misma cantidad de cables de una central de transmisión y el cliente final.

Así como hemos visto las técnicas anteriores se presentan inconvenientes a momento de su aplicación, pues atenuar o cancelar la diafonía requiere de una inversión muy costosa, por otro lado los resultados que se obtienen al utilizar estas técnicas no son del todo eficientes, nos vemos obligado a enfocar nuestro trabajo en una técnica que además de ser más factible su implementación en la práctica, no presenta ningún problema a nivel de costos, y además, lo más relevante de todo, es que los resultados que se obtienen de llevarla a cabo muestran un desempeño significante en la reducción de la diafonía, superando notablemente a las técnicas que se utilizan tanto en el lado del transmisor y en el lado del receptor, se trata de la técnica de cancelación ciega.

Cabe mencionar, que la técnica de cancelación ciega de la diafonía, la cual va a ser nuestro principal tema de estudio en este trabajo, se origina a partir del análisis de la cancelación ideal, técnica en la cual se requieren efectuar asunciones que luego se puede demostrar, pero la principal y más importante asunción que se hace se trata de tener el conocimiento explícito de la función de acoplamiento que se presenta entre todos los pares trenzados que se encuentran dentro del hilo de comunicación, función que en la práctica es compleja de obtener, es por esto, que en la cancelación ciega se realiza la estimación de la señal de diafonía en base a sus propiedades estadísticas, más que tener un conocimiento exacto de la función de acoplamiento, siendo esta aproximación por medio de propiedades estadísticas, muy útil, y muy cercana a la cancelación ideal, ya que si se conociera con exactitud los parámetros que se van a estimar, los resultados que se logran serían exactamente los mismos que en la técnica de la cancelación ideal de la diafonía.

1.2 Cancelación.

A continuación se analizará de manera detallada como se lleva a cabo la cancelación de la diafonía empezando primero por el estudio de la cancelación ideal, para luego finalizar con la técnica de cancelación ciega, la cual es el enfoque principal de nuestro trabajo.

Para esto, empezaremos definiendo el sistema en el que se va trabajar, el cual se trata del sistema de Línea Digital de Abonado, (DSL), en el cual vamos a explicar el diagrama de bloques con que trabaja y explicando además cada variable involucrada en el sistema para de esta manera tener una comprensión global de todo el análisis a realizarse.

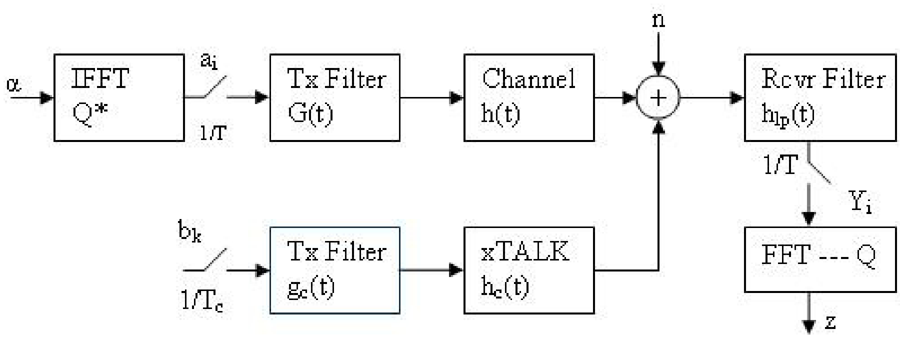

Figura 1.1 Modelo de un sistema de transmisión DMT con diafonía

El sistema descrito anteriormente, tiene como salida a la señal recibida en el dominio del tiempo identificada como y(t), la cual se describe a continuación.

$$y(t) = \sum_i a_i p(t - iT) + \sum_k b_k c(t - kT_c + \tau) + \tilde{n}(t)$$

Donde cada de una de las variables representadas en la expresión para $y(t)$ se describen a continuación:

a_i los símbolos de la señal DSL o también denotada como señal primaria.

b_k los símbolos de la señal de diafonía.

$\tilde{n}(t)$ es el ruido gaussiano, blanco, aditivo del canal.

$P(t)$ es la respuesta del filtro en el transmisor

$C(t)$ es la respuesta del crosstalk

T y T_c son los tiempos de muestreo el sistema DSL y de la diafonía respectivamente.

τ es el retardo entre una trama DMT y un símbolo de diafonía transmitido, donde ($0 < \tau < Tc$)

Cabe mencionar que la respuesta $P(t)$ está en función de la respuesta del filtro transmisor del sistema primario, $g(t)$, de la respuesta del canal del sistema primario, $h(t)$, y de la respuesta del filtro pasa bajos del receptor, $h_{lp}(t)$, así también $C(t)$ está en función de la respuesta del filtro transmisor del sistema interferente, $g_c(t)$, de la respuesta del canal del sistema de diafonía, $h_c(t)$, y de la respuesta del filtro pasa bajos del receptor, $h_{lp}(t)$.

$$p(t) = g(t) * h(t) * h_{lp}(t)$$

$$c(t) = g_c(t) * h_c(t) * h_{lp}(t)$$

Cabe recalcar que tanto la respuesta del sistema DSL o también denotado como primario, p(t), y la respuesta del sistema interferente o de diafonía, c(t), poseen respuestas de impulso finitas. Después de realizar el muestreo de la señal recibida, y(t), se obtiene la expresión detallada como sigue.

$$y_m = \sum_i a_i p((m-i)T) + \sum_k b_k c(mT - kT_c + \tau) + \tilde{n}_m$$

Como la información en los sistemas DMT es procesada en bloques de tamaño N, entonces N muestras de la expresión mencionada arriba pueden ser denotadas matricialmente como sigue:

$$Y = Pa + Cb + \tilde{n}$$

La información de entrada va a ser tratada en el dominio del tiempo, por lo que haciendo uso de la transformada inversa de Fourier, IFFT, la convertimos del dominio de la frecuencia al dominio del tiempo

$$a = Q^* \alpha$$

Donde Q es la matriz FFT, descrita matricialmente de la siguiente forma:

$$Q = \frac{1}{\sqrt{M}} \begin{bmatrix} e^{-j\frac{2\pi}{M}(M-1)(M-1)} & \cdots & e^{-j\frac{2\pi}{M}(M-1))} & 1 \\ e^{-j\frac{2\pi}{M}(M-2)(M-1)} & \cdots & e^{-j\frac{2\pi}{M}(M-2))} & 1 \\ e^{-j\frac{2\pi}{M}(M-1)} & \cdots & e^{-j\frac{2\pi}{M})} & 1 \\ 1 & \cdots & 1 & 1 \end{bmatrix}$$

El receptor usa ecualización para asegurarse de que la longitud del canal sea más corta que el prefijo cíclico, al ocurrir esto la matriz P se convierte en una matriz circulante y puede ser descompuesta como se detalla:

$$P = Q^{*}\Lambda Q$$

Donde Λ es una matriz diagonal, en la cual los elementos de la diagonal corresponden a la respuesta de frecuencia del canal.

$$\mathrm{diag}(\Lambda) = Q.\,[0 \ldots p_v \ldots p_0]^{*}$$

Siendo P como se describe a continuación, donde $v+1$ es la longitud del prefijo cíclico

$$P = \begin{bmatrix} p_0 & \cdots & p_{v-1} & p_v & 0 & \cdots & 0 \\ 0 & p_0 & \cdots & p_{v-1} & p_v & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 \cdots & 0 & p_0 & \cdots\ 0 & p_{v-1} & \cdots & 0 \\ p_v & 0 & \cdots\ 0 & p_0 & \cdots & & p_{v-1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ p_1 & \cdots & p_v & 0 & \cdots & 0 & p_0 \end{bmatrix}$$

La matriz de respuesta de la diafonía, C, es una matriz de dimensiones N x (L+μ), donde N es como ya hemos mencionado anteriormente el tamaño de las tramas DMT que se transmiten, L el número de símbolos de diafonía en una trama DMT, μ + 1 es el número de saltos cuando la señal de diafonía $c(t)$ es muestreada, siendo el elemento (i, j) de la matriz de diafonía el siguiente:

$$c_{ij} = c(\tau + (N - i)T - (L - j)T_c)$$

Para que se tenga una mejor comprensión de cómo está conformada la matriz de diafonía, se va a establecer que la señal primaria, denotada como a, va a tener M tonos, descritos como sigue $a=[a_{M-1}, a_{M-2}, \ldots, a_0]^T$, así también los símbolos de diafonía se definen como sigue $b=[b_{L-1}, \ldots, b_0, \ldots, b_{-\mu}]^T$, donde μ es el número de saltos de la respuesta del sistema de diafonía, y L es el número de símbolos de diafonía emitidos durante una trama DMT, parámetros que ya fueron definidos anteriormente.

Como se definió en la ecuación inmediata arriba mostrada, las filas de la matriz de diafonía van a estar gobernadas por el término que acompaña al tiempo de muestreo del sistema primario, T, es decir, en la primera fila irá el término (M – 1)T, en la segunda fila irá el término (M – 2)T, así hasta llegar a la última fila en la cual no habrá término, ya que el ultimo símbolo de la señal primaria está denotado como a_0, cabe mencionar que se usan los tonos de la señal primaria ya que se está multiplicando por el tiempo T, el cual es el tiempo de muestreo del sistema primario.

Así mismo, se va a proceder con la conformación de las columnas, las cuales van a estar regidas por el término que acompaña al tiempo de muestreo del sistema de diafonía o interferente, T_C, por lo que cabe recalcar que se va a trabajar con los símbolos de la diafonía, es decir, en la primera columna irá el término $(L-1)\ T_c$, en la segunda columna irá el término $(L-2)\ T_C$, así hasta llegar al término donde se utiliza el símbolo b_0, por lo que no habría término que acompañe a T_C , y tampoco habría el tiempo de muestreo Tc en esta columna, y así hasta llegar al último término el cual estaría definido por el ultimo símbolo de la señal de diafonía, el cual sería $b_{-\mu}$, por lo que μT_c sería el término de la última columna.

Habiendo finalizado la explicación de la constitución de la matriz de diafonía, C, la cual como se puede observar es de tamaño N x (L + μ), se la puede detallar como sigue.

$$C = \begin{bmatrix} c\left(\tau-(L-1)T_c+(M-1)T\right) & \cdots & c\left(\tau+(M-1)T\right) & \cdots & c\left(\mu T_c+\tau+(M-1)T\right) \\ c\left(\tau-(L-1)T_c+(M-2)T\right) & \cdots & c\left(\tau+(M-2)T\right) & \cdots & c\left(\mu T_c+\tau+(M-2)T\right) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c\left(\tau-(L-1)T_c+T\right) & \cdots & c(\tau+T) & \cdots & c\left(\mu T_c+\tau+T\right) \\ c\left(\tau-(L-1)T_c\right) & \cdots & c(\tau) & \cdots & c\left(\mu T_c+\tau\right) \end{bmatrix}$$

Habiendo descrito los principales elementos de nuestro sistema del lado del transmisor, ahora nos queda definir la señal recibida del lado del receptor, señal con la cual posteriormente vamos a trabajar para lograr el objetivo principal de nuestro trabajo, el cual es la eliminación de la diafonía.

De la misma manera que en el lado del transmisor, cuando se hizo la conversión de la señal DMT del dominio de la frecuencia al dominio del tiempo, el mismo trabajo se realiza en el lado del receptor, pero de manera inversa, esto con el fin de obtener la señal demodulada en el dominio de la frecuencia, esto lo obtenemos fácilmente multiplicando la señal recibida por la matriz de transformada de Fourier, FFT.

$z = Qy$

Usando los parámetros redefinidos que obtuvimos anteriormente para la matriz de respuesta del canal, podemos reemplazarlos en los valores establecidos inicialmente, además cabe mencionar que el ruido gaussiano blanco $n = Q * \tilde{n}$, tiene la misma varianza de ñ, por lo que la expresión de la señal demodulada quedaría definida como sigue.

$$z = \Lambda\alpha + QCb + Q\tilde{n}$$

Ver demostración 1.

Con la finalidad de obtener una mejor comprensión al final de esta sección en la que trataremos sobre el procedimiento paso a paso para llevar a cabo la cancelación de la diafonía, partiremos de la descripción de una cancelación de modo general, para luego seguir con el estudio de la cancelación ideal, para finalmente llegar a describir la técnica de la cancelación ciega, la cual es el enfoque principal de nuestro trabajo. Para esto comenzaremos redefiniendo la expresión de la señal demodulada, en el término correspondiente a la señal de diafonía, hacemos esto solo con el fin de conseguir describir una técnica de cancelación de forma general.

$$z = \Lambda\alpha + QCb + n$$

$$Hx = QCb$$

$$z = \Lambda\alpha + Hx + n$$

Partiendo de la expresión recientemente obtenida para la señal demodulada, subdividimos a dicha señal en dos grupos, que por motivos de facilidad de manejo matemático serán denotados vectorialmente.

$$\begin{bmatrix} z_2 \\ z_1 \end{bmatrix} = \begin{bmatrix} \Lambda_2\alpha_2 \\ \Lambda_1\alpha_1 \end{bmatrix} + \begin{bmatrix} H_2x \\ H_1x \end{bmatrix} + \begin{bmatrix} n_2 \\ n_1 \end{bmatrix}$$

Esta subdivisión de la señal demodulada en grupos se la realiza con el propósito de separar los elementos del exceso de banda del interferente de los elementos de la banda principal del interferente, por lo que el vector denotado como z_1 corresponde a los elementos pertenecientes al exceso de banda, y el vector denotado como z_2 corresponde a los elementos pertenecientes a la banda principal.

Siguiendo con el desarrollo de la técnica de cancelación, mencionamos que el vector correspondiente a los elementos del exceso de banda del interferente es donde se va a estimar la señal de diafonía, para luego con la ayuda de esta señal de diafonía estimada, denotada anteriormente como b, proceder con la construcción de la señal de cancelación, que la denotaremos como X_c, la cual es una señal que está en función de la matriz de transformada de Fourier, Q, la matriz de diafonía, C, y de la señal de diafonía, b, una vez obtenida esta señal cancelación, se procede a substraerla de la señal demodulada, consiguiendo con esto la eliminación de la diafonía.

Cabe mencionar que una asunción muy importante para poder realizar todo lo descrito anteriormente, tiene que ver con las tasas de muestreo, ya que se necesita imperiosamente que la tasa de muestro del sistema DSL sea mucho mayor que la tasa de muestreo del sistema de la diafonía, ya que esto permite que el sistema DSL pueda observar el exceso de banda del interferente, de igual manera destacar que la mayoría de los servicios más comunes de donde proviene la diafonía, tales como ISDN (Red Digital de Servicios Integrados), HDSL (Línea Digital de abonado de alta velocidad) y T1 suelen presentar grandes porcentajes de exceso de banda, lo cual nos facilita mucho más nuestro trabajo de cancelar la diafonía, ya que mientras más cantidades de exceso de banda presente el interferente, más cercana será la estimación de la señal de diafonía realizada.

Como siguiente paso asumimos que la señal demodulada en el grupo correspondiente al exceso de banda, puede ser detectada y descrita como $r = z_1 - \Lambda_1 \alpha_1$, ya que en este rango de frecuencias no se va a realizar la cancelación de la diafonía, sino más bien se la va a estimar, por lo que el vector r recién definido quedaría como sigue

$$r = H_1 x + n_1$$

Se define $\hat{x} = Mr$ es como una estimación lineal de x para cierta transformación lineal M, por lo que con la ayuda del Mean Squared Error se encuentra el estimador lineal para x, esto se realiza resolviendo el argumento del error entre la estimación de x y el valor real de x.

$$\arg\min_{M} E\left[(Mr - x)^{*}(Mr - x)\right]$$

La MMSE estimada para x es determinada por

$$\bar{M} = R_x H_1^* (H_1 R_x H_1^* + R_n)^{-1}$$

Ver Demostración 2.

Encontramos M

$$\bar{M} = (H_1^* R_n^{-1} H_1 + R_x^{-1})^{-1} H_1^* R_n^{-1}$$

Donde $R_x = E[xx^*]$ es la matriz de covarianza de la diafonía $R_n = E[nn^*]$ es la matriz de covarianza del ruido, como ya tenemos el valor de $\bar{M}$ podemos entonces obtener la estimación de x, la cual queda definida como se detalla a continuación.

La estimación de la señal de diafonía en el exceso de banda se describe a continuación

$$\hat{x} = Mr$$

$$\hat{x} = (H_1^* R_n^{-1} H_1 + R_x^{-1})^{-1} H_1^* R_n^{-1} r$$

Una vez obtenida la estimación de la diafonía en el exceso de banda, se realiza la construcción de la señal cancelación previamente descrita en la banda principal.

$$X_c = H_2 \hat{x}$$

$$X_c - H_2(H_1^* R_n^{-1} H_1 + R_x^{-1})^{-1} H_1^* R_n^{-1} r$$

Esta señal obtenida y definida como señal cancelación, no es nada más que la estimación de la diafonía en la banda principal, por lo que se realiza la substracción de la señal demodulada perteneciente a la banda principal, con lo cual quedaría por el momento concluido el trabajo de la cancelación de la diafonía.

$$z_2 - X_c = \Lambda_2 \alpha_2 + H_2 x + n_2 - X_c$$

$$z_2 - X_c = \Lambda_2 \alpha_2 + H_2(x - \hat{x}) + n_2$$

Cabe mencionar, que todo el procedimiento para cancelar la diafonía realizado anteriormente y las expresiones y estimaciones obtenidas, fueron tratadas como ya se lo explicó al inicio de esta sección, desde una perspectiva muy general, ya que esto nos sirve de base y de punto de partida para la mejor comprensión de las siguientes técnicas que vamos a estudiar en las secciones inmediatamente posteriores, como son las técnicas de cancelación ideal, y la técnica de cancelación ciega, la cual es el enfoque principal de nuestro trabajo.

1.2.1 Cancelación Ideal

Como se lo especificó previamente en la sección anterior, H y x fueron definidas así con el fin de al llegar al estudio tanto de la técnica de la cancelación como de la técnica de cancelación ciega, se pueda distinguir bien la diferencia básica que existe entre una técnica y otra, para poder así lograr una mejor visualización y un mejor entendimiento de la metodología de las mismas.

Por lo que para la cancelación ideal, técnica prioridad de análisis en esta sección, se define al parámetro H como función de la matriz de la transformada de Fourier, Q, y de la matriz de diafonía, C, en tanto que el parámetro x queda definido solo en función de la señal de diafonía, b.

$$H = Q^{*}C$$

$$x = b$$

Interpretando lo descrito anteriormente de forma matemática, decimos que en esta técnica el receptor conoce la función de acoplamiento, y se realiza la estimación de la señal de diafonía.

Una vez entendido esto, simplemente se reemplaza los valores de H y x con los recientemente definidos, en las expresiones obtenidas en la sección anterior, con lo que la señal cancelación perteneciente a la banda principal se define como sigue.

$$X_c = Q_2C(C^*Q_1^*R_n^{-1}Q_1C + R_b^{-1})^{-1}C^*Q_1^*R_n^{-1}r.$$

1.2.1.1 Consideración

Para implementar el algoritmo en la práctica es importante saber R_n y R_b por lo que asumimos que tanto la matriz de covarianza de la diafonía como la matriz de covarianza del ruido son blancos, y además que la señal de diafonía tiene varianza unitaria, por lo que redefinimos a la matriz de covarianza de la diafonía como $R_b = I$ y a la matriz de covarianza del ruido como $R_n = \sigma^2 I$ donde σ^2 es la varianza del ruido, por lo que la expresión obtenida para la señal cancelación queda definida por motivo de facilidad para la implementación como sigue.

$$X_c = Q_2C(C^*Q_1^*Q_1C + \sigma_n^2 I)^{-1}C^*Q_1^*r$$

Como ya se explicó anteriormente la matriz de diafonía, C, está en función del tiempo de retardo τ , el cual cambia de trama en trama, esto se produce debido a que la señal de entrada con la información a transmitirse, también llamada señal primaria, y la señal de diafonía tienen diferentes tasas de muestreo, por lo que se tiene que calcular esta matriz para cada trama DMT que se transmita, lo cual es muy complejo, por lo que se asume que se conoce la matriz de diafonía solo cuando τ es cero, esta asunción nos evita el calcular una matriz para cada bloque transmitido, esto se lleva a cabo por medio de aproximaciones basadas en el hecho de que un retardo en el dominio del tiempo es equivalente a un desplazamiento de fase en el dominio de la frecuencia, conociendo esto se pueden definir las siguientes aproximaciones.

$$Q_1 C \approx D_{\tau,1} Q_1 C_0$$

$$Q_2 C \approx D_{\tau,2} Q_2 C_0 \text{ , donde}$$

$$D_{\tau,1} = \mathrm{diag}\left(\left[e^{j2\pi\frac{k_1\tau}{NT}} \ldots e^{j2\pi\frac{k_m\tau}{NT}}\right]\right)$$

$$D_{\tau,2} = \mathrm{diag}\left(\left[e^{j2\pi\frac{l_1\tau}{NT}} \ldots e^{j2\pi\frac{l_m\tau}{NT}}\right]\right)$$

Donde C_0 es la matriz de diafonía cuando el tiempo de retardo τ es cero, $D_{\tau,1}$ es una matriz de fase con los componentes de desplazamiento de fase de la respuesta del canal de la diafonía, donde k_1, k_2, …, km son los tonos correspondientes al exceso de banda, $D_{\tau,2}$ es la matriz de fase con los componentes de desplazamiento de fase de la respuesta del canal de la diafonía, donde $l_1, l_2, \ldots, l_m$ son los tonos correspondientes a la banda principal, cabe mencionar que las aproximaciones arriba descritas pueden convertirse en exactas siempre y cuando el numero de tonos de las matrices de fase descritas sean bastante grandes.

Usando las aproximaciones detalladas, la expresión para la señal cancelación se la puede definir como sigue.

$$X_c = Q_2 C(C^* Q_1^* Q_1 C + \sigma_n^2 I)^{-1} C^* Q_1^* r$$

$$X_c \approx D_{\tau,2} \Phi_0 D_{\tau,1}^* r$$

$$\Phi_0 = Q_2 C_0 (C_0^* Q_1^* Q_1 C_0 + \sigma_n^2 I)^{-1} C_0^* Q_1^*$$

Ver Demostración 3.

Como se puede observar, la expresión Φ_0 es una función constante, ya que esta compuesta por valores que son conocidos, por lo que solamente necesitada ser calculada una vez, con esto se elimina el calculo de una matriz para cada bloque DMT transmitido, con lo que esta técnica se hace factible de implementar..

1.2.2 Cancelación Ciega

Tal como se lo hizo para la sección anterior, partimos de las expresiones obtenidas cuando se hizo el análisis de la cancelación de la diafonía de una forma general, distinguiendo así los valores de H y x, los cuales para esta técnica de cancelación ciega serán redefinidos, H en función de la matriz de transformada de Fourier, y x en función de la matriz de diafonía, C, y de la señal de diafonía.

$H = Q$

$x = Cb$

Definiendo esto, se interpreta que el receptor no tiene conocimiento de la función de acoplamiento, por lo que se tiene que estimar la matriz de diafonía y a la señal de diafonía juntas para poder llevar a cabo esta técnica de cancelación, reemplazando los parámetros redefinidos en esta sección en las expresiones obtenidas en el análisis de la cancelación en forma general, se construye la señal cancelación correspondiente a la banda principal, para poder así realizar la cancelación ciega de la diafonía.

$$\overline{M} = (Q_1^* R_n^{-1} Q_1 + R_{cb}^{-1})^{-1} Q_1^* R_n^{-1}$$

Estimamos los datos de la diafonía y la función de acoplamiento juntas

$$\widetilde{Cb} = (Q_1^* R_n^{-1} Q_1 + R_{cb}^{-1})^{-1} Q_1^* R_n^{-1} r$$

Lo que nos resulta la señal de cancelación

$$X_c = Q_2 (Q_1^* R_n^{-1} Q_1 + R_{cb}^{-1})^{-1} Q_1^* R_n^{-1} r$$

.

1.2.2.1 Consideración

Para empezar a analizar la implementación de la técnica de cancelación ciega es necesario mencionar que se va a utilizar parte de las asunciones descritas en la sección correspondiente a la implementación de la técnica de cancelación ideal, por lo que no está de más decir que la matriz de covarianza del ruido, R_n, será tratada como del tipo blanco.

Cabe mencionar además, que la técnica de cancelación ciega tiene el mismo inconveniente que la técnica de cancelación ideal, la cual se refiere al cálculo de una matriz que se tiene que realizar para cada bloque DMT transmitido, en este caso se debe a que la matriz de covarianza, R_{Cb}, está en función del tiempo de retardo tao, el cual varía su valor de trama en trama; por lo que como solución se va hacer uso de las aproximaciones obtenidas en la sección de la cancelación ideal correspondiente a la implementación, dicho esto no está de más volver a definir los parámetros que nos van a ayudar a eliminar el tener que calcular una matriz para cada trama DMT que se desee transmitir, por lo volvemos a detallar dichas funciones como sigue.

$$Q_i R_{cb} Q_j^* \approx D_i D_i R_{cb}(0) Q_j^* D_j^*$$

$$D_{\tau,1} = \mathrm{diag}\left(\left[e^{j2\pi\frac{k_1}{N}\frac{\tau}{T}} \ldots e^{j2\pi\frac{k_m}{N}\frac{\tau}{T}}\right]\right)$$

$$D_{\tau,2} = \mathrm{diag}\left(\left[e^{j2\pi\frac{k_1}{N}\frac{\tau}{T}} \ldots e^{j2\pi\frac{k_m}{N}\frac{\tau}{T}}\right]\right)$$

$$X_c = D_{\tau,2} Q_2 R_{Cb}(0) Q_1^* \left(Q_1 R_{Cb}(0) Q_1^* + D_{\tau,1}^* R_n D_{\tau,1}\right)^{-1} D_{\tau,1}^* r$$

Donde $R_{Cb}(0)$ es la matriz de covarianza de la matriz de diafonía y de la senal de diafonía, cuando el tiempo de retardo τ es cero, $D_{\tau,1}$ es una matriz de fase con los componentes de desplazamiento de fase de la respuesta del canal de la diafonía, donde k_1, k_2, …, km son los tonos correspondientes al exceso de banda, $D_{\tau,2}$ es la matriz de fase con los componentes de desplazamiento de fase de la respuesta del canal de la diafonía, donde l_1, l_2,…., lm son los tonos correspondientes a la banda principal, cabe mencionar que las aproximaciones arriba descritas pueden convertirse en exactas siempre y cuando el numero de tonos de las matrices de fase descritas sean bastante grandes

Ver Demostración 4

$$X_c = Q_2(Q_1^* R_n^{-1} Q_1 + R_{cb}^{-1})^{-1} Q_1^* R_n^{-1} r$$

$$X_c = D_{\tau,2} Q_2 R_{Cb}(0) Q_1^* \left(Q_1 R_{Cb}(0) Q_1^* + D_{\tau,1}^* R_n D_{\tau,1}\right)^{-1} D_{\tau,1}^* r$$

Ver Demostración 5

$$X_c \approx D_{\tau,2} Q_2 \Gamma_0 Q_1^* D_{\tau,1}^* r$$

Donde $\Gamma_0 = (Q_1^* Q_1 + \sigma_n^2 R_{Cb}^{-1}(0))^{-1}$ es una matriz constante, por lo cual solo se requiere que sea calculada una sola vez, con lo cual se elimina el cálculo de la matriz para cada trama DMT, además se puede aproximar estableciendo $\Phi_0 = Q_2 \Gamma_0 Q_1^*$, donde

$$\Phi_0 = Q_2 R_z(0) Q_1^* (Q_1 R_z(0) Q_1^*)^{-1}$$

Como se puede observar Φ_0 es una matriz constante, y no está en función de las matrices de covarianza de la matriz de diafonía y de la señal de diafonía, y de la matriz de covarianza del ruido, sino que depende ahora de la autocorrelación de la señal recibida o demodulada cuando el tiempo de retardo τ es cero, Rz(0), por lo que para la implementación de la técnica de cancelación ciega solo se necesita conocer la autocorrelación de la señal demodulada, $R_z(0)$, para poder construir la señal cancelación, lo cual no es del todo complejo, ya que se la puede estimar, incluso con mayor precisión cuando no exista trama DMT presente en la transmisión, o también cuando la señal se encuentre en un estado estable o estacionario; para realizar la estimación de la autocorrelación de la señal demodulada, $R_z(0)$, se asume primero que el tiempo de retardo τ para cada trama DMT transmitida es aproximadamente conocido, ya que se conocen las tasas de muestreo tanto del sistema de la señal primaria, como del sistema de la diafonía, teniendo conocimiento de este tiempo de retardo τ, la señal recibida puede ser desplazada de manera apropiada según el tiempo de retardo τ, que se presente para cada bloque DMT, con esto se puede calcular la autocorrelación de cada tono desplazado, cabe mencionar que estos desplazamientos resultan factibles realizarlos por medio de interpolación lineal, las matrices de autocorrelación calculadas en cada tono desplazado son promediadas para todo el número de bloques DMT transmitidos, con lo que se obtiene una buena estimación de la autocorrelación de la señal recibida cuando el tiempo de retardo τ es cero, $R_z(0)$, con lo que se puede realizar la construcción de la señal cancelación, con esto quedaría concluida la cancelación de la diafonía basándonos en el método ciego.

CAPÍTULO II.

2 DESARROLLO

2.1 Modelamiento

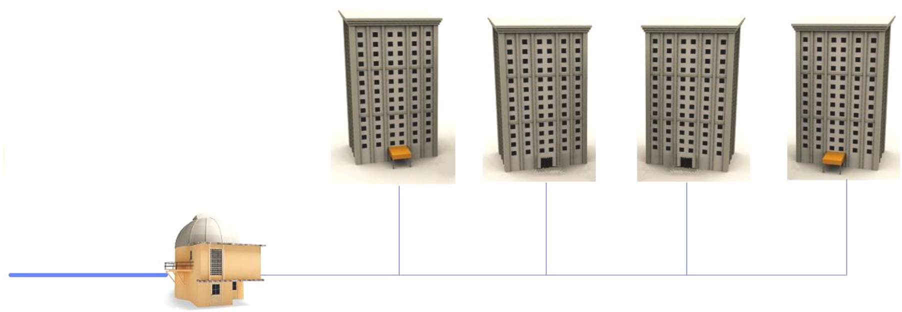

Figura 2.1 Grafica del ambiente de desarrollo

El sistema consiste en simular un canal VDSL2 de hasta 12MHz, en bland-plan 998, con profile 12ª, que contiene diafonía, el numero de perturbadores es de 40, el tipo de cable a usar en la simulación es TP150 0.5mm como lo detalla el estándar VDSL, la distancia máxima entre el cliente y la oficina central es de 1000 metros. Con una frecuencia de 25.87khz hasta 12 MHz

2.2 Simulación

En la siguiente grafica se muestra las condiciones y parámetros en que se va a trabajar la simulación.

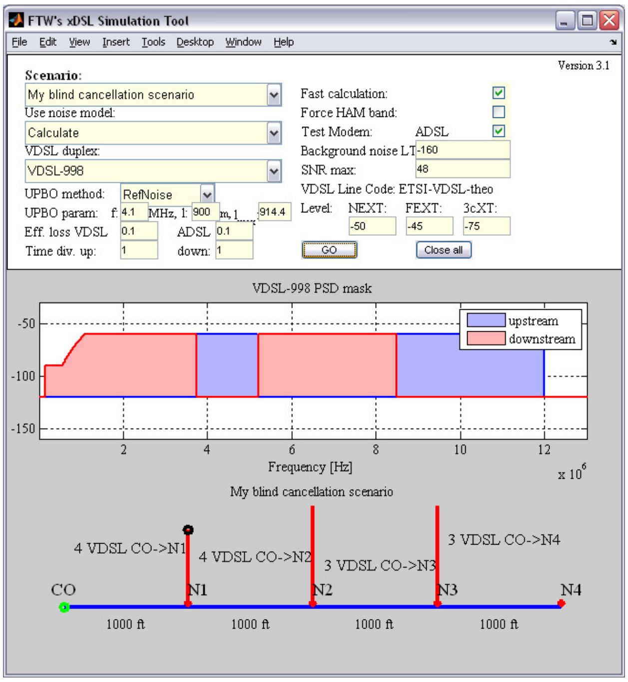

Figura 2.2 Modelo del escenario en MatLab

CAPÍTULO III.

3 ANALISIS DE LOS RESULTADOS

3.1 Análisis teórico

Basándonos en el análisis matemático extenso que se ha hecho a lo largo de este trabajo, se ha podido observar que tanto la técnica de cancelación ideal como la técnica de cancelación ciega resultan muy eficientes, ya que al realizar el análisis matemático de cada una de ellas se pudo constatar que aunque las dos técnicas presentan un poco de similitud en lo que es el desarrollo de la mecánica para proceder a llegar a la cancelación de la diafonía presente en el sistema, pero con la variante en los parámetros que se asumen conocer y en los parámetros que se van a estimar, pero con mejor resultado para la técnica de cancelación ciega sobre el método ideal, ya que en este último se asume el conocimiento de la función de acoplamiento entre los diferentes pares trenzados existentes, la cual como ya se mencionó es muy difícil de calcular, debido a esto y a otros aspectos que a continuación se detallarán, se pudo corroborar que el método de la cancelación ciega presenta resultados positivos en lo que es la cancelación de la diafonía, y además facilita la implementación de la misma ya que no se necesita conocer con exactitud la función de acoplamiento anteriormente mencionada.

En la técnica de cancelación ideal se puede destacar que el hecho de sólo estimar la diafonía, ya que se asume que se conoce la función de acoplamiento, en teoría facilita las cosas ya que sólo se tendría que trabajar con la autocorrelación de la señal de diafonía, R_b, pero no será así, ya que como bien sabemos la dificultad para conocer con exactitud la función de acoplamiento, el cual es el mayor y principal inconveniente para que no se opte por implementar en la práctica esta técnica.

Se puede corroborar en nuestro trabajo desarrollado, que el método ciego es mas practico que el ideal por el motivo que la función de acoplamiento es muy compleja de calcular en la práctica, tal como fue explicado, por lo que nuestro método se hace mas practico ya que no se necesita el conocimiento de dicha función sino mas bien la estimación de la misma, resultando un proceso menos complejo.

Los beneficios que nos ofrece la cancelación de diafonía en términos numéricos han sido demostrados [2] en un ambiente ADSL, proporcionando una mejor área de cobertura a los clientes con un aumento del 30 al 70% y un incremento de la tasa de datos entre dos y tres veces su valor inicial, dependiendo de las características del servicio del que proviene la diafonía. Con estos resultados podemos aplicar a una cancelación en cualquier sistema xDSL a que sabemos que los resultados obtenidos mejoran nuestro servicio.

CONCLUSIONES

1. Se puede decir que la cancelación por el método ciego, presenta mejores resultados en servicios que presentan un alto porcentaje de exceso de banda, lo cual, es muy útil para nuestro método, ya que en estos rangos de frecuencia se realiza la estimación de la diafonía.

2. Se puede concluir que el efecto de cancelar la diafonía refleja una mejora en la relación señal ruido, la cual aumenta cuando los interferentes son mitigados, produciendo así un aumento en la tasa de datos, aprovechando el canal completamente para una mayor cantidad de bits transmitidos, motivo por el cual ha sido desarrollado este trabajo.

3. RECOMENDACIONES

1. Para quienes en un futuro deseen estudiar y analizar los distintos métodos existentes para la cancelación o atenuación de la diafonía, no solo el método ciego, sino cualquiera de los que ya han sido mencionados al inicio de nuestro trabajo, podemos decir que hay que tener bien claro la parte matemática y estadística, para obtener una comprensión más rápida de lo que involucra las propiedades estadísticas de las señales, tanto de las señales que contienen la información, denominadas como primarias, como de las señales interferentes, las cuales son las que producen la diafonía en nuestro sistema de comunicación, ya que sin la comprensión de lo recién mencionado sería casi imposible conseguir un entendimiento global de este problema.

ANEXOS

Anexo A Direcciones electrónicas de sitios Web

http://www.mathworks.es/ , Junio del 2010

http://es.wikipedia.org/wiki/MATLAB, Junio del 2010

http://es.wikipedia.org/wiki/Diafon%C3%ADa, Junio del 2010

http://en.wikipedia.org/wiki/Asymmetric_digital_subscriber_line, Junio del 2010

http://www.zonagratuita.com/servicios/noticias/2005/junio/009.htm, Junio del 2010

Anexo B Demostraciones

Demostración 1:

$$z = Qy$$
$$z = Q(Pa + Cb + \tilde{n})$$
$$z = Q(PQ^{*}\alpha + Cb + \tilde{n})$$
$$z = Q(Q^{*}\Lambda QQ^{*}\alpha + Cb + \tilde{n})$$
$$z = Q(Q^{*}\Lambda QQ^{*}\alpha + Cb + \tilde{n})$$
$$z = \Lambda\alpha + QCb + Q\tilde{n}$$

Demostración 2:

$$\bar{M} = \mathrm{R}_x H_1^{*}(H_1 R_x H_1^{*} + R_n)^{\ 1}$$
$$= \mathrm{R}_x H_1^{*}\left((H_1^{*})^{-1} R_x{}^{-1} H_1^{*} + R_n{}^{-1}\right)$$

$= \mathrm{R}_x H_1^*(H_1^*)^{-1} R_x{}^{-1}(H_1^*)^{-1} + R_x H_1^* R_n{}^{-1}$
$= \mathrm{R}_x R_x{}^{-1}(H_1^*)^{-1} + R_x H_1^* R_n{}^{-1}$
$= H_1^{-1} + R_x H_1^* R_n{}^{-1}$
$= H_1^{-1} R_n (H_1^*)^{-1} H_1^* R_n + R_x H_1^* R_n{}^{-1}$
$= (H_1^{-1} R_n (H_1^*)^{-1} + R_x) H_1^* R_n{}^{-1}$
$\overline{M} = (H_1^* R_n{}^{-1} H_1 + R_x{}^{-1}) H_1^* R_n{}^{-1}$

Demostración 3:

$X_c = Q_2 C (C^* Q_1^* C Q_1 C + \sigma_n^2 I)^{-1} C^* Q_1^* r$
$Q_1 C \approx D_{\tau,1} Q_1 C_0$
$Q_2 C \approx D_{\tau,2} Q_2 C_0$
$= D_{\tau,2} Q_2 C_0 (C_0^* Q_1^* D_{\tau,1}^* D_{\tau,1} Q_1 C_0 + \sigma_n^2 I)^{-1} C_0^* Q_1^* D_{\tau,1}^* r$
$= D_{\tau,2} Q_2 C_0 (C_0^* Q_1^* Q_1 C_0 + \sigma_n^2 I)^{-1} C_0^* Q_1^* D_{\tau,1}^* r$
$X_c = D_{\tau,2} \Phi_0 D_{\tau,1}^* r$
$Q_2 C_0 (C_0^* Q_1^* Q_1 C_0 + \sigma_n^2 I)^{-1} C_0^* Q_1^* \text{ es contante} \rightarrow \Phi_0$

Demostración 4:

$X_c = Q_2 (Q_1^* R_n^{-1} Q_1 + R_{cb}^{-1})^{-1} Q_1^* R_n^{-1} r$
Donde, $Q_1 R_{Cb} Q_j^* \approx D_1 Q_i R_{Cb}(0) Q_j^* D_j^*$
$X_c = Q_2 (Q_1^{-1} R_n (Q_1{}^*)^{-1} + R_{cb})\ Q_1^* R_n^{-1} r$
$X_c = (Q_2 Q_1^{-1} R_n (Q_1{}^*)^{-1} + Q_2 R_{cb})\ Q_1^* R_n^{-1} r$
$X_c = Q_2 Q_1^{-1} R_n (Q_1{}^*)^{-1} Q_1^* R_n^{-1} r + Q_2 R_{cb} Q_1^* R_n^{-1} r$
$X_c = Q_2 Q_1^{-1} r + Q_2 R_{cb} Q_1^* R_n^{-1} r$
$X_c = Q_2 R_{cb} Q_1^* (Q_1^*)^{-1} R_{cb}^{-1} Q_1^{-1} r + D_{\tau,2} Q_2 R_{cb}(0) Q_1^* D_{\tau,1}^* R_n^{-1} r$
$X_c = D_{\tau,2} Q_2 R_{cb}(0) Q_1^* D_{\tau,1}^* (Q_1^*)^{-1} R_{cb}^{-1} Q_1^{-1} r + D_{\tau,2} Q_2 R_{cb}(0) Q_1^* D_{\tau,1}^* R_n^{-1} r$
$X_c = D_{\tau,2} Q_2 R_{cb}(0) Q_1^* (D_{\tau,1}^* (Q_1^*)^{-1} R_{cb}^{-1} Q_1^{-1} D_{\tau,1} D_{\tau,1}^* r + D_{\tau,1}^* R_n^{-1} D_{\tau,1} D_{\tau,1}^* r)$
$X_c = D_{\tau,2} Q_2 R_{cb}(0) Q_1^* (D_{\tau,1}^* (Q_1^*)^{-1} R_{cb}^{-1} Q_1^{-1} D_{\tau,1} + D_{\tau,1}^* R_n^{-1} D_{\tau,1}) D_{\tau,1}^* r$
$X_c = D_{\tau,2} Q_2 R_{cb}(0) Q_1^* (D_{\tau,1}^* (Q_1^{-1})^* R_{cb}^{-1} Q_1^{-1} D_{\tau,1} + D_{\tau,1}^* R_n^{-1} D_{\tau,1}) D_{\tau,1}^* r$
$X_c = D_{\tau,2} Q_2 R_{cb}(0) Q_1^* (D_{\tau,1}^* (D_{\tau,1}^{-1})^* (Q_1^{-1})^* R_{cb}^{-1}(0) Q_1^{-1} D_{\tau,1}^{-1} D_{\tau,1} + D_{\tau,1}^* R_n^{-1} D_{\tau,1}) D_{\tau,1}^* r$
$X_c = D_{\tau,2} Q_2 R_{cb}(0) Q_1^* (D_{\tau,1}^* (D_{\tau,1}^{-1})^* (Q_1^{-1})^* R_{cb}^{-1}(0) Q_1^{-1} D_{\tau,1}^{-1} D_{\tau,1} + D_{\tau,1}^* R_n^{-1} D_{\tau,1}) D_{\tau,1}^* r$
$X_c = D_{\tau,2} Q_2 R_{cb}(0) Q_1^* ((Q_1^{-1})^* R_{cb}^{-1}(0) Q_1^{-1} + D_{\tau,1}^* R_n^{-1} D_{\tau,1}) D_{\tau,1}^* r$
$X_c = D_{\tau,2} Q_2 R_{cb}(0) Q_1^* ((Q_1^*)^{-1} R_{cb}^{-1}(0) Q_1^{-1} + D_{\tau,1}^* R_n^{-1} D_{\tau,1}) D_{\tau,1}^* r$

$$X_c = D_{\tau,2}Q_2R_{cb}(0)Q_1^*(Q_1R_{cb}(0)Q_1^* + D_{\tau,1}^*R_n\, D_{\tau,1})^{-1}D_{\tau,1}^*r$$

Demostración 5:

$$X_c = D_{\tau,2}Q_2R_{cb}(0)Q_1^*\left(Q_1R_{cb}(0)Q_1^* + D_{\tau,1}^*R_nD_{\tau,1}\right)^{-1}D_{\tau,1}^*r$$
$$X_c = D_{\tau,2}Q_2R_{cb}(0)Q_1^*\left((Q_1^*)^{-1}R_{cb}^{-1}(0)Q_1^{-1} + D_{\tau,1}^*R_n^{-1}(D_{\tau,1}^*)^{-1}\right)D_{\tau,1}^*r$$
$$X_c = D_{\tau,2}Q_2\left(R_{cb}(0)Q_1^*(Q_1^*)^{-1}\cancel{R_{cb}^{-1}}\cancel{(0)}Q_1^{-1} + R_{cb}(0)Q_1^*D_{\tau,1}^*R_n^{-1}(D_{\tau,1}^*)^{-1}\right)D_{\tau,1}^*r$$
$$X_c = D_{\tau,2}Q_2\left(R_{cb}(0)R_{cb}^{-1}\cancel{(0)}\cancel{Q_1^{-1}} + R_{cb}(0)Q_1^*D_{\tau,1}^{-1}R_n^{-1}(D_{\tau,1}^*)^{-1}(Q_1^*)^{-1}Q_1^*\right)D_{\tau,1}^*r$$
$$X_c = D_{\tau,2}Q_2\left(Q_1^{-1}(Q_1^*)^{-1}Q_1^* + R_{cb}(0)Q_1^*D_{\tau,1}^{-1}R_n^{-1}(D_{\tau,1}^*)^{-1}(Q_1^*)^{-1}Q_1^*\right)D_{\tau,1}^*r$$
$$X_c = D_{\tau,2}Q_2\left(Q_1^{-1}(Q_1^*)^{-1}Q_1^* \underline{+R_{cb}}(0)Q_1^*D_{\tau,1}^{-1}R_n^{-1}(D_{\tau,1}^*)^{1}(Q_1^*)^{-1}\underline{Q_1^*)D}_{\tau,1}^*r$$
$$X_c = D_{\tau,2}Q_2\left(Q_1^{-1}(Q_1^*)^{-1} + R_{cb}(0)Q_1^*D_{\tau,1}^{-1}R_n^{-1}(D_{\tau,1}^*)^{-1}(Q_1^*)^{-1}\right)(Q_1^*D_{\tau,1}^*r)$$
$$X_c = D_{\tau,2}Q_2\left(Q_1^*Q_1 + Q_1^*D_{\tau,1}^*R_nD_{\tau,1}(Q_1^*)^{-1}R_{cb}^{-1}(0)\right)^{-1}(Q_1^*D_{\tau,1}^*r)$$
$$X_c = D_{\tau,2}Q_2\left(Q_1^*Q_1 + Q_1^*D_{\tau,1}^*\sigma_n^2ID_{\tau,1}(Q_1^*)^{-1}R_{cb}^{-1}(0)\right)^{-1}(Q_1^*D_{\tau,1}^*r)$$
$$X_c = D_{\tau,2}Q_2\left(Q_1^*Q_1 + \sigma_n^2Q_1^*D_{\tau,1}^*\cancel{D_{\tau,1}}\cancel{(Q_1^*)}^{-1}R_{cb}^{-1}(0)\right)^{-1}(Q_1^*D_{\tau,1}^*r)$$
$$X_c = D_{\tau,2}Q_2\left(Q_1^*Q_1 + \sigma_n^2Q_1^*\cancel{(Q_1^*)}\cancel{^{-1}R}_{cb}^{-1}(0)\right)^{-1}(Q_1^*D_{\tau,1}^*r)$$
$$X_c = D_{\tau,2}Q_2\left(Q_1^*Q_1 + \sigma_n^2R_{cb}^{-1}(0)\right)^{-1}(Q_1^*D_{\tau,1}^*r)$$

Donde, $\left(Q_1^*Q_1 + \sigma_n^2R_{cb}^{-1}(0)\right)^{-1} = \Gamma_0$

$$\rightarrow X_c = D_{\tau,2}Q_2\Gamma_0Q_1^*\mathrm{D}_{\tau,1}^*r$$

Anexo C Papers

PALABRAS CLAVE

NEXT	(Near-end Cross Talk). Es la fracción de señal que aparece en el extremo cercano de un par adyacente
FEXT	(Far-end Cross Talk). Fracción de señal que aparece en el extremo opuesto de un par adyacente.
xDSL	Cualquiera de los diversos tipos de tecnologías de línea de abonado digital de banda ancha.
VDSL2	Very High Speed Digital Suscriber Line Transceiver 2, nueva tecnología Banda ancha que se redacto en la norma ITU-T G.993.2.

Blind Crosstalk Cancellation for DMT Systems

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Abstract—As the deployment of broadband communication systems such as DSL continues to grow, system performance in terms of capacity and error rates is severely limited by crosstalk interference. In order to continue deploying high speed DSL service, it becomes necessary to mitigate this crosstalk interference. All current crosstalk mitigation techniques require precise knowledge of the crosstalk coupling functions between the twisted pair wires carrying DSL service within a cable binder. In practice, this is near impossible to obtain, without coordination between the different DSL services. In this paper, we present a crosstalk cancellation technique that is blind to the coupling functions between the wire pairs. Our technique uses the statistical properties of the received crosstalk signal (that can be easily estimated), rather than the exact coupling functions themselves, making it very attractive for practical implementation. We show with the help of simulations for a realistic ADSL system with HDSL and T1 crosstalk interference using actual measured crosstalk coupling functions that the proposed blind crosstalk cancellation technique can achieve significant gains in terms of rate and reach improvement for the ADSL system. We also show that the performance of the proposed blind crosstalk cancellation technique is robust to a jitter in the crosstalk symbol timing estimate that is required to construct the cancellation signal.

Keywords—crosstalk, NEXT, mitigation, blind, cancellation.

I. Introduction

New multimedia and networking applications keep increasing end users demand for bandwidth. The network backbone of access providers generally consists of high-speed fiber optic links. Cost prevents these links from reaching end users. In recent years interim solutions such as Digital Subscriber Lines (DSL) provide broadband access to end users without expensive infrastructure upgrades. DSL techniques provide high speed network access over ordinary twisted pair telephone wires by extending the range of usable frequencies. Ordinary voice calls use a bandwidth of up to 4 Khz, while with DSL the range of usable frequencies is extended to several MHz.

Typically twisted pair wires carrying service from the central office (CO) to remote terminals (RTs) are packed closely together, up to 50 at a time, within cable binders. The proximity of these wires leads to crosstalk coupling between them, which often increases with frequency. Hence, crosstalk significantly limits the achievable bitrates of such high bandwidth, high speed systems [1].

Several approaches can be taken to combat the effects of crosstalk. Transmitter side approaches often work by shaping the spectra [5] or block coding at the transmitter [6] to avoid crosstalk rather than cancel it. In practice transmitter crosstalk mitigation techniques are difficult to implement as they require changes in existing standards. Other receiver side approaches use decision feedback equalization [4] for crosstalk cancellation. Multiuser detection techniques [8] and their blind variations, could possibly be used for crosstalk cancellation. However, we are not interested in decoding all the crosstalk signals, and moreover these techniques are computationally expensive. Most existing crosstalk mitigation techniques require explicit knowledge of the coupling functions between wire pairs. In practice, this is almost impossible to obtain without coordination across RTs and at the CO, or some third party monitoring all data transmission and relaying the coupling functions to hardware. Such systems would be very complex and would require changes to current standards. A crosstalk mitigation technique that is blind to the coupling functions, relatively easy to implement and provides reasonable performance is highly desirable, and is the focus of this paper.

The outline for the paper is as follows, section II provides some background and notation, III describes the crosstalk cancellation problem and the proposed blind crosstalk cancellation solution, IV describes details for implementation, V some simulation results and finally VI provides some points for discussion and conclusions.

II. Background and Notation

Crosstalk can be categorized into one of two forms. The first, Near End Crosstalk (NEXT) is interference arising when signals are transmitted in opposite directions, and Far End Crosstalk (FEXT), when signals are transmitted in the same direction.

NEXT is far more damaging due to the proximity of the interfering transmitter and receiver while FEXT signals propagate the length of the line and are significantly attenuated. Crosstalk can be caused by interferers of the same type of service or interferers from other services.

In this paper, we consider cancelling crosstalk from services like HDSL, T1 and ISDN that have large excess bandwidths. For the purposes of our discussion, we consider DMT (discrete multitone modulation) based ADSL, but the results equally applicable to other DMT services.

We now provide a summary of the notation, keeping it consistent with [2]. The DSL system environment can be described as follows

$$y(t) = \sum_i a_i p(t - iT) + \sum_k b_k c(t - kT_c + \tau) + \tilde{n}(t) \tag{1}$$

where a_i and b_k are the DSL and crosstalk data, and $\tilde{n}(t)$ is white gaussian noise. The effective DSL and crosstalk pulse shapes, including transmit and receive filters, are $p(t)$ and $c(t)$, respectively. The sampling periods of the DSL and crosstalk systems are T and T_c, and τ is the fractional timing difference between the start of the DMT frame and the crosstalk symbol. The receiver samples (1) with frequency $\frac{1}{T}$ to get

$$y_m = \sum_i a_i p((m - i)T) + \sum_k b_k c(mT - kT_c + \tau) + \tilde{n}_m \tag{2}$$

In DMT based systems data is processed in blocks of size N. Therefore, N samples from (2) are buffered leading to a compact matrix notation

$$\mathrm{y} = P\mathrm{a} + C\mathrm{b} + \tilde{\mathrm{n}} \tag{3}$$

Data is modulated and demodulated via the fast Fourier transform (FFT) and its inverse (IFFT) operation. Let Q denote the FFT matrix. Data modulation is denoted

$$\mathrm{a} = Q^*\alpha \tag{4}$$

where α and a are the frequency and time domain data, before and after modulation. DMT systems use a cyclic prefix (CP), a periodic extension of the DMT block, to combat interchannel and intersymbol interference. The receiver uses equalization to ensure that the channel is shorter than the CP. When this occurs the channel matrix, P, becomes circulant, and can be decomposed as

$$P = Q^*\Lambda Q \tag{5}$$

with Λ being a diagonal matrix

$$diag(\Lambda) = Q \cdot [0 \ldots p_\nu \ldots p_0]^* \tag{6}$$

where $\nu+1$ is the CP length. C is an $N \times (L+\mu)$ crosstalk matrix, where the $(i,j)^{th}$ element is

$$c_{ij} = c\left(\tau + (N - i)T - (L - j)T_c\right) \tag{7}$$

and $\mu + 1$ is the number of taps when $c(t)$ is sampled. L is the number of crosstalk symbols in a single DMT frame. The effective crosstalk signal delay, τ, changes from block to block as the symbol rates of the DSL and crosstalk system are not the same. At the other end of the system, the receiver performs the FFT operation on (2).

$$\begin{aligned} \mathrm{z} &= Q\mathrm{y} \\ &= \Lambda\alpha + QC\mathrm{b} + Q\tilde{\mathrm{n}} \\ &= \Lambda\alpha + QC\mathrm{b} + \mathrm{n} \end{aligned} \tag{8}$$

III. Crosstalk Cancellation

A. Problem Setup

Let us begin with the demodulated vector

$$\mathrm{z} = \Lambda\alpha + H\mathrm{x} + \mathrm{n} \tag{9}$$

with Λ, α and n defined previously and $H\mathrm{x} = QC\mathrm{b}$. The demodulated vector can be divided into two groups

$$\begin{bmatrix} \mathrm{z}_2 \\ \mathrm{z}_1 \end{bmatrix} = \begin{bmatrix} \Lambda_2\alpha_2 \\ \Lambda_1\alpha_1 \end{bmatrix} + \begin{bmatrix} H_2\mathrm{x} \\ H_1\mathrm{x} \end{bmatrix} + \begin{bmatrix} \mathrm{n}_2 \\ \mathrm{n}_1 \end{bmatrix} \tag{10}$$

with elements subscripted by 1, the set where crosstalk will be estimated and those subscripted by 2 the set where crosstalk will be cancelled. A possible grouping of elements is to separate those in the excess band from those in the main band of the intereferer [2].

We assume that the sampling rate, $1/T$, of the DSL system is higher than that of the crosstalk system, $1/T_c$. Therefore the crosstalk is oversampled, allowing the DSL system to see the excess band of the crosstalker. DSL services like HDSL, ISDN and T1 typically have transmit filters that gradually rolloff. This results in a significant amount of transmit (image) energy in the excess band.

Next, we assume that we can reliably detect the DSL signal the excess band, that is we are able to correctly demodulate α_1. This is a reasonable assumption as DMT based DSL systems like ADSL typically operate with some excess noise margin (i.e., 6dB noise margin over a target bit error rate of 1e-7).

Further no crosstalk cancellation is assumed in the excess band, hence the error in demondulating α_1 is negligible. Now let

$$\begin{aligned} \mathrm{r} &= \mathrm{z}_1 - \Lambda_1\alpha_1 \\ &= H_1\mathrm{x} + \mathrm{n}_1 \end{aligned} \tag{11}$$

Let $\hat{\mathrm{x}} = M\mathrm{r}$ be a linear estimate of x, for some linear transformation M. Then the error is $(\hat{\mathrm{x}} - \mathrm{x}) = (M\mathrm{r} - \mathrm{x})$ and the mean square error is $E[(M\mathrm{r}-\mathrm{x})^*(M\mathrm{r}-\mathrm{x})]$. The linear MMSE estimate for x is determined by solving

$$\arg\min_M E\left[(M\mathrm{r} - \mathrm{x})^*(M\mathrm{r} - \mathrm{x})\right] \tag{12}$$

The random vector r and the inner product $E[\cdot]$ form a Hilbert space, allowing us to use the orthognality principle to find the M which achieves a minimum

$$\bar{M} = R_x H_1^* \left(H_1 R_x H_1^* + R_n\right)^{-1} \tag{13}$$

where $R_x = E[\mathrm{x}\mathrm{x}^*]$ and $R_n = E[\mathrm{n}\mathrm{n}^*]$. This can also be shown to be equivalent to

$$\bar{M} = \left(H_1^* R_n^{-1} H_1 + R_x^{-1}\right)^{-1} H_1^* R_n^{-1} \tag{14}$$

The estimate of the crosstalk signal is then

$$\begin{aligned}\widehat{\mathrm{x}} &= \bar{M}\mathrm{r} \\ &= \left(H_1^* R_n^{-1} H_1 + R_x^{-1}\right)^{-1} H_1^* R_n^{-1}\mathrm{r}\end{aligned} \tag{15}$$

and the projection onto the main band

$$\begin{aligned}X_c &= H_2\widehat{\mathrm{x}} \\ &= H_2\left(H_1^* R_n^{-1} H_1 + R_x^{-1}\right)^{-1} H_1^* R_n^{-1}\mathrm{r}\end{aligned} \tag{16}$$

We use this estimate to subtract the crosstalk from the main band.

$$\begin{aligned}\mathrm{z}_2 - X_c &= \Lambda_2\alpha_2 + H_2\mathrm{x} + \mathrm{n}_2 - X_c \\ &= \Lambda_2\alpha_2 + H_2(\mathrm{x} - \widehat{\mathrm{x}}) + \mathrm{n}_2\end{aligned} \tag{17}$$

The resulting signal can then be demodulated for α_2. The amount of reduction in crosstalk energy in the main band directly determines the performance gain obtained.

B. *Ideal Crosstalk Cancellation*

We now specify variables from the previous section. Setting $H_1 = Q_1 C$ and $\mathrm{x} = \mathrm{b}$, we arrive at the same solution as described in [2]. Hence, [2] is a special case of the general solution described in III-A. In this case the cancellation signal becomes

$$X_c = Q_2 C\left(C^* Q_1^* R_n^{-1} Q_1 C + R_b^{-1}\right)^{-1} C^* Q_1^* R_n^{-1}\mathrm{r} \tag{18}$$

C. *Blind Crosstalk Cancellation*

The setup in section III-A, solves the problem in the general sense, while section III-B describes the case when the receiver knows the crosstalk coupling function. To make the algorithm blind to the couplings function, we set $H_1 = Q_1$ and $\mathrm{x} = \mathrm{Cb}$. The matrix $\bar{M}$ thus becomes

$$\bar{M} = \left(Q_1^* R_n^{-1} Q_1 + R_{Cb}^{-1}\right)^{-1} Q_1^* R_n^{-1} \tag{19}$$

and we estimate the crosstalk data and coupling function together as

$$\widehat{\mathrm{Cb}} = \left(Q_1^* R_n^{-1} Q_1 + R_{Cb}^{-1}\right)^{-1} Q_1^* R_n^{-1}\mathrm{r} \tag{20}$$

which results in the cancellation signal

$$X_c = Q_2\left(Q_1^* R_n^{-1} Q_1 + R_{Cb}^{-1}\right)^{-1} Q_1^* R_n^{-1}\mathrm{r} \tag{21}$$

In comparing the schemes in III-B and III-C we see they differ only in their assignment of the H_1 and x variables from III-A. The ideal technique assumes that the receiver has knowledge of the coupling function and assigns $H_1 = Q_1 C$ and estimates the crosstalk data $\mathrm{x} = \mathrm{b}$. The blind approach assumes that the coupling function C is unknown to the receiver. It assigns $H_1 = Q_1$ and estimates $\mathrm{x} = \mathrm{Cb}$ the crosstalk and coupling function together.

IV. Implementation Details

A. *Ideal*

To implement the algorithm in practice we must have knowledge of R_n and R_b. As in [2] we can assume $R_b = I$ and $R_n = \sigma^2 I$. The crosstalk cancellation signal becomes

$$X_c = Q_2 C\left(C^* Q_1^* Q_1 C + \sigma_n^2 I\right)^{-1} C^* Q_1^*\mathrm{r} \tag{22}$$

We notice that matrix C is a function of τ, the relative delay between the DMT frame and the crosstalk symbol which changes from block to block. Any implementation requires us to find C for each DMT block and perform a matrix inversion, which is prohibitvely complex.

In [2], the author proposes a solution for the first problem. If C is known only for $\tau = 0$, then we can approximate $Q_1 C \approx D_{\tau,1} Q_1 C_0$ and $Q_2 C \approx D_{\tau,2} Q_2 C_0$ where

$$D_{\tau,1} = diag\left(\left[e^{j2\pi\frac{k_1}{N}\frac{\tau}{T}} \ldots e^{j2\pi\frac{k_m}{N}\frac{\tau}{T}}\right]\right) \tag{23}$$

is a phase shift matrix, and $k_1, \ldots, k_m$ are the tone indices in the excess band. We define $D_{\tau,2}$ similarly, for tones in the main band. Using these approximations, the crosstalk cancellation signal becomes

$$X_c \approx D_{\tau,2}\Phi_0 D_{\tau,1}^*\mathrm{r} \tag{24}$$

where $\Phi_0 = Q_2 C_0\left(C_0^* Q_1^* Q_1 C_0 + \sigma_n^2 I\right)^{-1} C_0^* Q_1^*$ is constant and needs to be computed only once. This avoids a matrix inversion for each block and allows the algorithm to be computationally feasible. However, the outstanding issue of determining the crosstalk coupling function, C_0, remains to be solved.

B. *Blind*

The solution to the blind crosstalk cancellation problem in (21) requires R_{Cb} which is a function of τ, the relative crosstalk symbol delay that changes every DMT frame. Rather, we would like to use R_{Cb} for $\tau = 0$, to avoid matrix inversions for each DMT frame. Using $D_{\tau,1}$ and $D_{\tau,2}$ as defined earlier, it is straightforward to show that

$$Q_i R_{Cb} Q_j^* \approx D_i Q_i R_{Cb}(0) Q_j^* D_j^* \tag{25}$$

Beginning with equation (21) and using the above approximation we get

$$\begin{aligned}X_c \approx D_{\tau,2} Q_2 R_{Cb}(0) Q_1^* &\left(Q_1 R_{Cb}(0) Q_1^*\right. \\ &\left.+ D_{\tau,1}^* R_n D_{\tau,1}\right)^{-1} D_{\tau,1}^*\mathrm{r}\end{aligned} \tag{26}$$

and if we assume white noise, $R_n = \sigma^2 I$, and use the equivalence between equations (13) and (14) we arrive at

$$X_c \approx D_{\tau,2} Q_2 \Gamma_0 Q_1^* D_{\tau,1}^*\mathrm{r} \tag{27}$$

where $\Gamma_0 = \left(Q_1^* Q_1 + \sigma_n^2 R_{Cb}^{-1}(0)\right)^{-1}$ is a constant matrix and only needs to be computed once, avoiding matrix inversion from block to block. In fact $\Phi_0 = Q_2 \Gamma_0 Q_1^*$. Using (13) and (14) we arrive at

$$\Phi_0 = Q_2 R_z(0) Q_1^* (Q_1 R_z(0) Q_1^*)^{-1} \qquad (28)$$

so we don't need R_{Cb} and R_n individually but instead $R_z(0)$, the autocorrelation of the demodulated vector.

B.1 Estimating Correlation

The blind cancellation method requires $R_z(0)$ to construct the cancellation signal. It is simple and accurate to estimate this autocorrelation during quiet periods in training when no DMT signal is present or in a decision directed manner in steady state.

The relative delay between the crosstalk symbols and the start of the DMT block varies from block-to-block, and needs to be estimate for each block. We show that the overall performance is robust to an error in estimating this relative delay. Assuming that the delay is known (within some accuracy), the received vector can be shifted appropriately and the autocorrelation of each shifted waveform computed. Shifting can be accomplished using linear interpolation. The autocorrelation matrices produced are averaged together for a large number of DMT blocks, resulting in an accurate estimate for $R_z(0)$.

V. Simulation Results

We simulated the performance of the proposed blind crosstalk cancellation scheme for a realistic ADSL system, with HDSL and T1 NEXT interferers and an AWGN floor of $-140dBm/Hz$. Our simulations use actual measured crosstalk coupling functions for a 24 wire pair binder. Performance is measured in the downstream direction from the central office to the remote user. The downstream band extends from 25.875KHz to 1.104MHz (tones 6 to 255 with a spacing of 4.3125kHz), as is the case for an echo cancelled ADSL modem.

Figure 1 shows the performance of the blind and ideal crosstalk cancellation methods in cancelling HDSL NEXT. The ADSL system is operating at a critically sampled rate of $f_s = 2.208$Mhz. The main band for the HDSL crosstalk, extends from 0-192kHz (Nyquist frequency is 192kHz) and the excess band, for estimating the crosstalk signal, extends from 192-384kHz. In figure 1 for the blind method, we have shown performance with an exact timing estimate (of the crosstalk symbol delay relative to the start of the DMT block) and also with an approximate timing estimate. We have considered two cases for the approximate timing estimate where the crosstalk symbol timing is known to within an accuracy of (a) $\frac{1}{4}^{th}$ the DMT sampling period, denoted by Q(T/4) and (b) $\frac{1}{2}$ the DMT sampling period, denoted by Q(T/2).

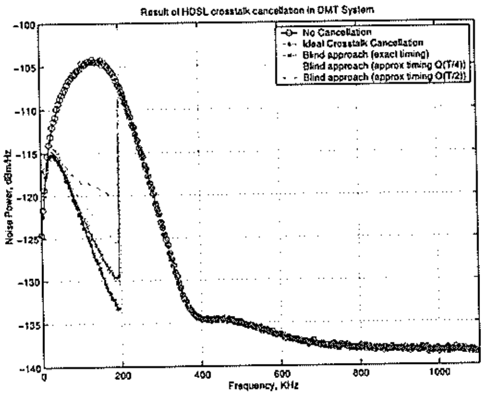

Fig. 1. HDSL NEXT cancellation for ideal and blind approaches.

From the figure, we observe that with exact knowledge of the symbol timing the performance of the blind method is comparable to that of the ideal method that knows the exact coupling function. The slight deviation in performance between the ideal and blind approaches results from the fact that the blind approach uses an estimate for $R_z(0)$ (using the phase shift matrices) while the ideal approach uses the exact crosstalk coupling matrix $C0$, rather than an estimate. If the blind method was to use the *exact* R_z, then the performance is identical to the ideal method. Further, even in the realistic scenario of an approximate symbol timing estimate, we observe a significant reduction in the crosstalk signal power in the main band ie., a 15dB and 12dB reduction of average crosstalk signal energy is observed for the Q(T/4) and Q(T/2) timing cases.

The corresponding rate vs. reach curves for the ADSL system are shown in figure 2. We observe that for deployment data rates of 1Mbps the proposed blind crosstalk cancellation method extends ADSL system reach by 1500ft and 1000ft (for Q(T/4) and Q(T/2) cases).

Next we consider a situation with combined HDSL and T1 crosstalk signals. We apply the crosstalk cancellation technique successively. First we cancel T1 crosstalk, as it has a wider bandwidth (Nyquist frequency is 772KHz) and then cancel HDSL crosstalk. We assume the realistic scenario that both crosstalkers are continuously present in the system even during training of the crosstalk canceller (better performance will be obtained if the crosstalk cancellation matrices for HDSL and T1 are separately estimated for the two individual crosstalkers). The main and excess bands for the T1 crosstalk signal are assumed to be 0-772kHz and 772-1544kHz. It is obvious that the ADSL receiver needs to operate at a 2x oversampled rate in order to estimate the T1 crosstalk signal from its excess band.

Figure 3 shows the performance of the proposed blind crosstalk cancellation technique (with both exact and ap-

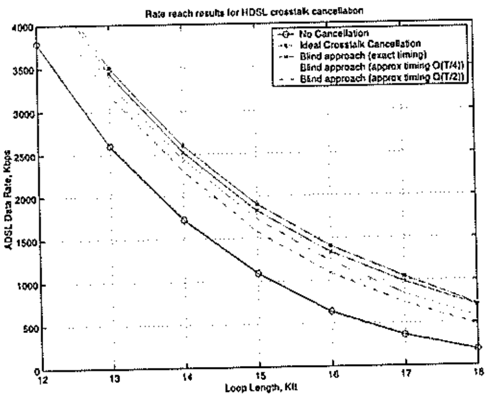

Fig. 2. Rate vs. reach curve for ADSL system with HDSL NEXT cancellation

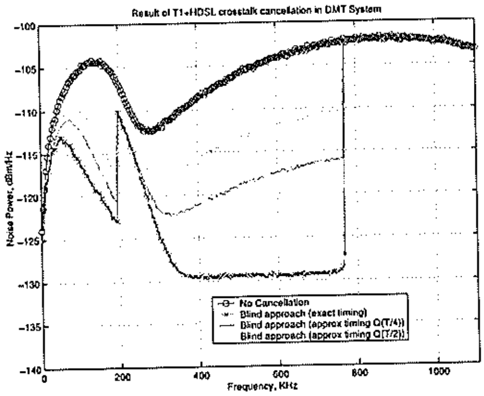

Fig. 3. HDSL and T1 combined NEXT cancellation for ideal and blind approaches.

proximate timing estimates) in a scenario with combined HDSL and T1 crosstalk. We observe a significant reduction in the crosstalk signal power in the main band ie., a 12dB and 7dB energy reduction for the Q(T/4) and Q(T/2) timing cases. The corresponding rate vs. reach curves for the ADSL system are shown in figure 4. For deployment data rates of 1Mbps our method extends ADSL system reach by 2000ft and 1500ft (for Q(T/4) and Q(T/2) cases).

There are two reasons for the lower amount of cancelled crosstalk energy (12dB and 7dB) in this case: (a) an increased noise floor; T1 increases the noise floor when cancelling HDSL and vice versa and (b) a greater effect of timing jitter; the effect of timing jitter is proportional to frequency and hence affects T1 cancellation to a much larger extent than HDSL cancellation. However, the bandwidth over which T1 energy is cancelled is much larger than the HDSL case. Hence, the net effect in terms of rate-reach performance improvement is greater in the

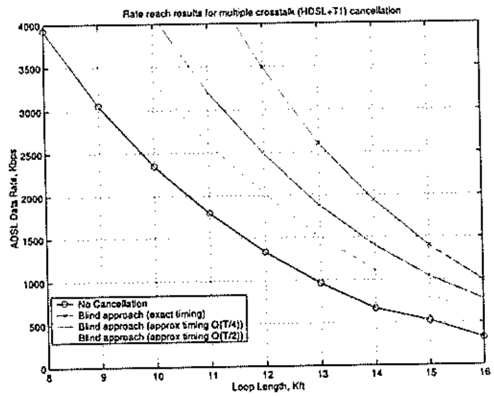

Fig. 4. Rate vs. reach curve for ADSL system with combined HDSL and T1 NEXT cancellation

combined T1 + HDSL case than in the pure HDSL case.

VI. Discussion and Conclusions

Crosstalk interference is a limiting factor for DSL system performance. Many techniques have been proposed to combat its effects. However, to the best of our knowledge, no solutions exist that effectively cancel crosstalk without explicit knowledge of the coupling functions between twisted-pair wires within a cable binder.

In this paper, we have proposed a blind crosstalk cancellation method for DMT systems that is based entirely on the crosstalk signal statistics (not requiring knowledge of the actual crosstalk coupling functions). Our solution has low complexity making it practical to implement while also achieving reasonably good performance. Our proposed blind crosstalk cancellation method is also robust to a jitter in the crosstalk symbol timing estimate.

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Crosstalk Cancellation in xDSL Systems

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Abstract

Near-end crosstalk (NEXT) is one of the major impairments to the current ADSL downstream transmission. This paper presents two methods for an ADSL receiver to cancel one (dominant) NEXT signal from other types of services (such as HDSL, SDSL, T1, etc). The methods exploit the fact that the crosstalk signal has a large excess bandwidth and its spectra in the main lobe and in the excess band are strongly correlated. The principal idea is then to estimate the crosstalk in some frequency bands (e.g., excess band) and cancel it in other frequency bands (e.g., main lobe). The frequency-domain analysis in this paper provides an intuitive explanation of the crosstalk estimation and cancellation, as well as a guidance to select the right frequency bands to observe the crosstalk signal. Moreover, a fast algorithm is proposed for practical implementation. This algorithm avoids matrix inversion and large matrix multiplication in every transmission block. Simulation results show that one of the proposed methods, MMSE estimation and cancellation, is very effective to cancel one (dominant) NEXT and the improvement is significant in terms of the data rate and the line reach for the ADSL service. For example, using a real measured NEXT transfer function, the proposed method can increase the ADSL downstream data rate by 200% for some loops. The methods are extended to estimate and cancel two or more crosstalkers. The amount of improvement depends on the crosstalkers' characteristics and it is generally less than that of a single crosstalker case.

Keywords

DSL, crosstalk, excess bandwidth, decision-aided cancellation, MMSE estimation and cancellation.

I. Introduction

Digital subscriber line (DSL) technology uses the existing phone lines to offer high speed data transmission services to both residential and business customers. There are many types of DSLs [1], generically referred to as xDSL, including basic rate DSL (ISDN), HDSL (high-bit-rate DSL), ADSL (asymmetric DSL), HDSL2 (second generation HDSL), SDSL (single-pair, symmetric DSL), and VDSL (very-high-bit-rate DSL). Of these DSLs, ISDN, ADSL, and HDSL have been standardized by International Telecommunication Union (ITU). ITU-T Recommendation G.995.1 [3] provides a comprehensive overview of these standardized recommendations. HDSL2 and VDSL are currently in the process of being standardized. SDSL is not standardized but has been deployed to offer various data rate less than 1.536Mbps.

One of the major impairments of the xDSL systems is the severe crosstalk [2] among the telephone lines in the same or neighboring bundles. The crosstalk is classified into near-end crosstalk (NEXT) and far-end crosstalk (FEXT). In general, NEXT is much larger than FEXT because the interference source is closer to the receiver. Therefore, ADSL and VDSL use frequency division duplexing (FDD) to avoid NEXTs from the same services. However, other types of services (such as HDSL, SDSL, T1, etc.), which use different duplexing schemes and overlap in frequency with ADSL and VDSL, may produce detrimental NEXT. Mitigating the effect of NEXT in an ADSL receiver can dramatically increase the data rate, the line reach, or the system operational margin.

The optimum detector for the interference cancellation can be theoretically achieved by maximizing the *a posteriori* probability of the primary signal (MAP rule), which is unfortunately too complex in practice. Some suboptimal multiuser detectors [4][5][6][7] are proposed to mitigate or cancel the interference signal in the non-spreading system. These detectors decode each user's data using "soft" symbols and iterate the detection process until a certain criterion is reached (e.g., the maximum number of iterations). The convergence of this type of algorithms is an open problem. The algorithms are still very complex when the signals have large constellation sizes [7].

This paper presents new practical methods to cancel or mitigate one (dominant) NEXT for an ADSL receiver. The principal idea is to estimate the crosstalk signal in certain frequency bands and subtract it in other frequency bands. A similar idea [8][9] has been previously used to suppress *very* narrow band radio frequency interference (RFI) in VDSL systems. The crosstalk signal to an ADSL receiver has large excess bandwidth and its spectra in the main lobe and the excess band are strongly correlated, which gives the opportunity to cancel the crosstalk signal in some dependent frequency bands. For example, the crosstalk can be estimated in the excess band and cancelled in the main lobe; vice versa. This paper provides a guidance on how to select the best frequency bands to observe the crosstalk signal and an intuitive interpretation of the crosstalk cancellation process. Another important aspect of the proposed techniques is that they can be implemented with low computational complexity, without matrix inversion or large matrix multiplication in each transmission block.

Previously, the fractionally-spaced equalizer (FSE) was used to suppress cyclostationary NEXTs [10][11] if both the crosstalk signals and the primary signal are synchronized and have excess bandwidth. The FSE processes the signals' spectrum in both the main band and the excess band. The folded spectrum after resampling to the symbol rate then provides the flexibility to suppress NEXTs in the main band. The problem addressed in this paper is different mainly in the following two aspects. First, the primary signal (ADSL) and the crosstalk signal (such as NEXT from HDSL, SDSL, T1) have completely different modulation schemes and sampling rates. Second, the primary received signal is decoded in the frequency domain, thus the crosstalk signal suppression is also processed in the frequency domain. In fact, the frequency domain explanation in this paper gives an insight on how much NEXT can be suppressed.

This paper proceeds as follows. Section II describes the system model of the primary and crosstalk channels. Section III presents the methods to cancel the crosstalk signal and a fast computation scheme for practical implementation. The methods are then extended to estimate and cancel two or more crosstalkers. Simulation results are shown in Section IV to verify the proposed methods. Section V concludes the paper.

In this paper, the notations are arranged in the following convention. A small letter, a bold small letter, and a capital letter represent a scalar, a vector, and a matrix, respectively. The superscript symbols ^{T} and * represent "transpose" and "conjugate and transpose" operations, respectively.

II. System Model

ADSL [12] uses the discrete multiple tone (DMT) modulation scheme [13][14] for data transmission. DMT is an effective realization of multicarrier transmission [15][16][17], which partitions the intersymbol interference (ISI) channel into a large number of narrowband subchannels. There is no or little ISI in each subchannel if the bandwidth of the subchannel is sufficiently narrow. The data is then transmitted in each subchannel almost free of ISI. A subchannel is more often called a "tone" in DMT systems and this terminology will be used in the rest of this paper.

The crosstalk signal from HDSL, SDSL, T1, or ISDN has different modulation schemes. HDSL, SDSL, and ISDN use 2B1Q baseband transmission and T1 uses alternative mark inversion (AMI) baseband transmission.

Fig. 1 shows a general model of a primary DMT transmission system with one crosstalker.

respectively. The DMT signal has a cyclic prefix ($a_{M-i} = a_{-i}$, $i = 1, \cdots, \nu$), therefore the channel response matrix P is circulant and has the following form,

$$P = \begin{bmatrix} p_0 & \cdots & p_{\nu-1} & p_\nu & 0 & \cdots & 0 \\ 0 & p_0 & \cdots & p_{\nu-1} & p_\nu & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & p_0 & \cdots & p_{\nu-1} & p_\nu \\ p_\nu & 0 & \cdots & 0 & p_0 & \cdots & p_{\nu-1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ p_1 & \cdots & p_\nu & 0 & \cdots & 0 & p_0 \end{bmatrix}$$

where $\nu + 1$ is the number of taps of the channel response $p(t)$. The crosstalk matrix $C_{M\times(L+\mu)}$ is

$$C = \begin{bmatrix} c(\tau - (L-1)T_c + (M-1)T) & \cdots & c(\tau + (M-1)T) & \cdots & c(\mu T_c + \tau + (M-1)T) \\ c(\tau - (L-1)T_c + (M-2)T) & \cdots & c(\tau + (M-2)T) & \cdots & c(\mu T_c + \tau + (M-2)T) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c(\tau - (L-1)T_c + T) & \cdots & c(\tau + T) & \cdots & c(\mu T_c + \tau + T) \\ c(\tau - (L-1)T_c) & \cdots & c(\tau) & \cdots & c(\mu T_c + \tau) \end{bmatrix}$$

where $\mu + 1$ is the number of taps of the crosstalk response, L is the number of crosstalk symbols in one DMT block ($L = \lceil MT/T_c \rceil$). Note that there are many zero entries in the above matrix C. Since the delay τ changes block over block, the matrix C varies over different blocks.

With a cyclic prefix, the circulant matrix P can always be decomposed [18, p. 201-2] as

$$P = Q^* \Lambda Q \tag{6}$$

where Q is a fast Fourier transform (FFT) matrix, Λ is a diagonal matrix whose diagonal elements correspond to the frequency response of the channel. More specifically, the FFT matrix is

$$Q = \frac{1}{\sqrt{M}} \begin{bmatrix} e^{-j\frac{2\pi}{M}(M-1)(M-1)} & \cdots & e^{-j\frac{2\pi}{M}(M-1)} & 1 \\ e^{-j\frac{2\pi}{M}(M-2)(M-1)} & \cdots & e^{-j\frac{2\pi}{M}(M-2)} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j\frac{2\pi}{M}(M-1)} & \cdots & e^{-j\frac{2\pi}{M}} & 1 \\ 1 & \cdots & 1 & 1 \end{bmatrix}$$

and the diagonal elements of Λ from the top left to the right bottom are

$$diag(\Lambda) = Q \cdot \begin{bmatrix} 0 \\ \vdots \\ p_\nu \\ \vdots \\ p_0 \end{bmatrix}.$$

At the transmitter, the signal $\boldsymbol{\alpha}$ is modulated in the frequency domain and transformed into the time domain by an inverse FFT for transmission, i.e., $\mathbf{a} = Q^*\boldsymbol{\alpha}$. At the receiver, the received signal is transformed back to the frequency domain. Therefore, the whole system model in the frequency domain is

$$\mathbf{z} = Q\mathbf{y} = \Lambda\boldsymbol{\alpha} + QC\mathbf{b} + \mathbf{n}. \tag{7}$$

where $\mathbf{z} = [z_{M-1}, z_{M-2}, \cdots, z_0]^T$, and the white Gaussian noise $\mathbf{n} = Q\tilde{\mathbf{n}}$ has the same variance as $\tilde{\mathbf{n}}$. The traditional system treats the crosstalk signal as Gaussian interference, which significantly limits the overall system performance. This paper presents new methods to cancel or suppress the crosstalk component $QC\mathbf{b}$ in the received signal. The channel and crosstalk responses $p(t)$ and $c(t)$ are assumed to be known in the ADSL receiver. The channel response is obtained through training sequences and the crosstalk response can be acquired by the method in [20].

III. Crosstalk Cancellation

The principal idea of crosstalk cancellation in this section is to first estimate the crosstalk signal by observing the output of certain frequency bands and then reconstruct the crosstalk signal in other bands to cancel out the interference. If the crosstalk signal has an excess bandwidth, like HDSL and SDSL crosstalkers in the xDSL systems, the cancellation is possible because the excess bandwidth provides more information about the crosstalk signal. In this section, the basic idea to cancel the crosstalk signal is described under the terminology of the above system model. A geometrical interpretation is introduced to explain the cancellation process intuitively and to select the right frequency bands to observe the crosstalk signal. Then, a fast computational method is proposed for practical implementation, which avoids the matrix inversion in the crosstalk signal estimation and cancellation. Finally, the method is extended to more than one crosstalker.

A. Crosstalk Signal Estimation and Cancellation

ADSL uses DMT modulation and the received signal is processed in the frequency domain. In order to detect and cancel the crosstalk signal, all the tones of interest[1] are partitioned into two disjoint sets, S_1 and S_2. In set S_1, the DMT system treats the crosstalk signal as a Gaussian noise, like a traditional system. The primary signal is detected and subtracted from the received signal. Then the crosstalk signal $\mathbf{b}$ is estimated by observing the residual signal in set S_1. With the estimated crosstalk signal $\tilde{\mathbf{b}}$, the interference in set S_2 can be constructed and subtracted from the received signal. If the interference in set S_2 can be completely eliminated, then the primary DMT system will have a significantly higher signal to interference and noise ratio (SINR) and more bits can be transmitted in this set of tones. The channel model representation in (7) is re-grouped according to the partition:

$$\begin{bmatrix} \mathbf{z}_2 \\ \mathbf{z}_1 \end{bmatrix} = \begin{bmatrix} \Lambda_2 \boldsymbol{\alpha}_2 \\ \Lambda_1 \boldsymbol{\alpha}_1 \end{bmatrix} + \begin{bmatrix} Q_2 C \mathbf{b} \\ Q_1 C \mathbf{b} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_2 \\ \mathbf{n}_1 \end{bmatrix} \tag{8}$$

where every vector or matrix with the subscript belongs to the set with the same subscript. For example, $\mathbf{z}_1$ and $\mathbf{z}_2$ are the received data in sets S_1 and S_2, respectively. Assume the primary signal in set S_1 can be detected reliably and denote $\tilde{\mathbf{z}}_1 = \mathbf{z}_1 - \Lambda_1 \boldsymbol{\alpha}_1$, then crosstalk signal can be estimated by a linear minimum mean-squares error (MMSE) estimator [19, p. 95] as

$$\tilde{\mathbf{b}} = (R_{\mathbf{b}}^{-1} + C^* Q_1^* R_{\mathbf{n}}^{-1} Q_1 C)^{-1} C^* Q_1^* R_{\mathbf{n}}^{-1} \tilde{\mathbf{z}}_1 \tag{9}$$

[1] Only includes those tones where the crosstalk signal exists, not all the tones in the ADSL receiver.

where R_{b} and R_{n} are the signal and noise covariance matrices respectively. The transmitted crosstalk signal sequence $\mathbf{b}$ and the background noise are normally assumed to be white, and the crosstalk signal can be assumed to have unit variance without loss of generality. Then the above equation can be further simplified to

$$\tilde{\mathbf{b}} = (\sigma_n^2 I + C^* Q_1^* Q_1 C)^{-1} C^* Q_1^* \tilde{\mathbf{z}}_1 \tag{10}$$

where σ_n^2 is the background noise variance. There are several different approaches to construct the crosstalk signal in set S_2. Here are the two simple approaches.

1. Linear MMSE estimation and cancellation. The receiver directly uses the estimated signal from (10) to construct the interference signal:

$$\begin{aligned} X_c &= Q_2 C \tilde{\mathbf{b}} \\ &= Q_2 C (\sigma_n^2 I + C^* Q_1^* Q_1 C)^{-1} C^* Q_1^* \tilde{\mathbf{z}}_1. \end{aligned} \tag{11}$$

and subtracts it from the received signal in set S_2. In fact, $\tilde{\mathbf{b}}$ can be considered as a special kind of "soft" decision for interference cancellation. The error covariance matrix of the estimated crosstalk signal from (10) is

$$\begin{aligned} \epsilon_b &= E(\tilde{\mathbf{b}} - \mathbf{b})(\tilde{\mathbf{b}} - \mathbf{b})^* \\ &= \sigma_n^2 (\sigma_n^2 I + C^* Q_1^* Q_1 C)^{-1}. \end{aligned} \tag{12}$$

Then the error covariance of the constructed interference is

$$\epsilon_X = \sigma_n^2 Q_2 C (\sigma_n^2 I + C^* Q_1^* Q_1 C)^{-1} C^* Q_2^*. \tag{13}$$

2. Decision-aided cancellation. The receiver makes "hard" decision on $\tilde{\mathbf{b}}$ to decode the transmitted crosstalk signal in set S_1, and constructs the interference signal in set S_2 based on the decision. The disadvantage is that a decision error would double the detrimental impact on the constructed interference. The improvement can be achieved by making "soft" decision with extra computational complexity.

The first approach works better if the transmitted crosstalk signal can not be reliably decoded. For example, the HDSL crosstalker signal includes the low-frequency band $[0 - 26kHz]$, which is eliminated by the front-end filter of the ADSL receiver. As a result, the

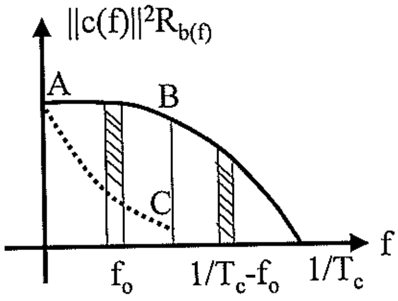

Fig. 2. **Crosstalk estimation and cancellation.**

transfer function for the crosstalk signal has a null response in this band, which would produce many incorrect decisions if the crosstalk signal were decoded. The decision-aided cancellation, however, works better if the interference to noise ratio in set S_1 is relatively high such that most of the decisions are correct. Inherently, this approach takes advantage of the known discrete constellation of the crosstalk source signal.

B. Geometrical Interpretation and Tone Selection

A natural question arises on how to select the best frequency bands to estimate the crosstalk signal and to mitigate the interference in the other bands. The answers for the two approaches above will be described separately in the following. The crosstalk signal uses baseband transmission, therefore the signal in the negative frequency is a conjugate of the counterpart in the positive frequency. For simplicity, this paper describes the method in the single-sided positive frequency band.

B.1 MMSE estimation and cancellation

The right strategy, which will be justified later, is as follows:

1. calculate SINRs in frequencies f_o and $1/T_c - f_o$, where T_c is the crosstalk signal sampling period;
2. estimate the crosstalk signal in the frequency bin that has a smaller SINR;
3. cancel the interference in the other bin that has the larger SINR.

In Fig. 2, the solid curve shows the power spectral density (PSD) of the crosstalk signal, with 100% excess bandwidth. The crosstalk signal is estimated in the excess band and then used to cancel the interference in the main lobe $(0, 1/2T_c)$. The dotted curve $\overline{AC}$ shows the residual crosstalk PSD as a result of the crosstalk subtraction. If the smaller one of the interference to noise ratios in frequencies f_o and $1/T_c - f_o$ is denoted by g_s (the shaded zone in the figure), then the SINR gain in frequency f_o is approximately equal to g_s. Similarly, if the crosstalk signal is estimated in the main lobe and used to cancel the interference in the excess band, the SINR gain in frequency $1/T_c - f_o$ is also approximately equal to g_s. These statements will be proved later in this subsection.

In fact, the gain is the same no matter which frequency, either f_o or $1/T_c - f_o$, is used to estimate the crosstalk signal. However, the data rate increase is not the same because it depends on the original SINR. For example, if the original SINR in frequency $1/T_c - f_o$ is $-9dB$ and the gain resulting from crosstalk cancellation is also $9dB$, then the capacity is increased by about $0.5bits/s/Hz$; if the original SINR in frequency f_o is $10dB$ and the gain is the same, then the capacity is increased by about $1.5bits/s/Hz$. Therefore, the crosstalk signal should be estimated in frequency $1/T_c - f_o$ and cancelled in frequency f_o. Otherwise, the data rate improvement is smaller ($0.5b/s/Hz$ versus $1.5b/s/Hz$). This phenomenon could actually happen in an ADSL environment. When the line reach is very long, the ADSL signal in the excess band is weak compared to the NEXT signal, therefore, the crosstalk signal should be estimated in the excess band and cancelled in the main lobe. Correspondingly, if the primary signal has a smaller SINR in the main lobe for some reason (e.g., because of the bridge tap), the crosstalk signal should be first estimated in the main lobe and cancelled in the excess band.

The following provides mathematically the rationale of the above statements. The problem is easier to understand in the frequency domain instead of the matrix representation in (11). The short-term Fourier transform (STFT) of the system model in (1) is defined as

$$
\begin{aligned}
y(f) &= \frac{1}{\sqrt{MT}} \int_0^{MT} y(t) e^{-j2\pi f t} dt \\
&\approx p(f)a(f) + c(f)e^{j2\pi f \tau} b(f) + n(f)
\end{aligned}
\tag{14}
$$

where $p(f)$ and $c(f)$ are the normal frequency responses of $p(t)$ and $c(t)$, respectively. The noise $n(f)$ is a stochastic process, which is the STFT of $\tilde{n}(t)$. The variance of $n(f)$ is equal to its PSD, i.e., $E(n(f)n^*(f)) = \sigma_n^2$. The approximation in (14) is due to the term of the crosstalk signal, because the crosstalk signal does not have the cyclic prefix and is not cyclostationary. This approximation is accepted for the following three reasons. First, the approximation error is small if M is relatively large and is exact if $M \to \infty$. Second, the analysis resulting from this approximation is used only for a guidance to select the right frequency bins to observe the crosstalk signal, not for the real crosstalk estimation. Third, this approximation makes the analysis much more succinct and provides an intuitive explanation of the estimation and cancellation process.

The primary and the crosstalk signals are

$$a(f) = \frac{1}{\sqrt{MT}} \sum_i a_i e^{-j2\pi f T i},$$
$$b(f) = \frac{1}{\sqrt{MT}} \sum_k b_k e^{-j2\pi f T_c k}.$$

The crosstalk signal has the following property: $b(f) = b^*(1/T_c - f)$, where T_c is the crosstalk symbol period. Therefore, if the crosstalk component in frequency $1/T_c - f_o$ is observed, the interference component in frequency f_o can be reconstructed.

In the following analysis, $0 < f_o < 1/(2T_c)$ is assumed without loss of generality. The crosstalk signal b_k is assumed to be white and is uncorrelated with the primary signal a_i. For brevity, symbol f_d denotes frequency $1/T_c - f_o$. The linear MMSE estimation of $b(f_d)$, given the observation $y(f_d)$, is

$$\tilde{b}(f_d) = \frac{c^*(f_d)e^{-j2\pi f_d \tau} R_b}{||c(f_d)||^2 R_b + \sigma_n^2} (y(f_d) - p(f_d)a(f_d)) \tag{15}$$

where $R_b = E(b_k b_k^*)L/(MT) \approx \varepsilon_b / T_c$, and $||c(f_d)||^2 R_b$ is the PSD of the transmitted crosstalk signal at frequency f_d. The reconstructed interference in frequency f_o is then

$$x_c(f_o) = c(f_o) e^{j2\pi f_o \tau} \tilde{b}^*(f_d). \tag{16}$$

The variance of the cancellation residual error plus the background noise is

$$\sigma_t^2(f_o) = \sigma_n^2 \frac{||c(f_o)||^2 R_b}{||c(f_d)||^2 R_b + \sigma_n^2} + \sigma_n^2. \tag{17}$$

The original error variance without cancellation is

$$\sigma_x^2(f_o) = ||c(f_o)||^2 R_b + \sigma_n^2. \tag{18}$$

Therefore, the SINR gain is equal to

$$\begin{aligned} Gain &= \frac{\sigma_x^2(f_o)}{\sigma_t^2(f_o)} \\ &= 1 + \frac{||c(f_o)||^2||c(f_d)||^2 R_b^2}{\sigma_n^2(||c(f_o)||^2 R_b + ||c(f_d)||^2 R_b + \sigma_n^2)} \qquad (19) \\ &= 1 + \frac{PSD(f_o) \cdot PSD(f_d)}{\sigma_n^2\,(PSD(f_o) + PSD(f_d) + \sigma_n^2)} \qquad (20) \end{aligned}$$

where $PSD(f_o)$ is the interference PSD at frequency f_o.

Proposition 1: Suppose the PSDs of the primary signal and the background noise are fixed. Given the dual frequencies f_o and f_d, the SINR gain by the MMSE estimation and cancellation is the same no matter which frequency is used for crosstalk estimation. However, with respect to the data rate improvement, the crosstalk should be estimated in the frequency with a lower SINR and cancelled in its dual frequency.

Proof: The SINR gain in (20) is derived by estimating the crosstalk signal in frequency f_o and cancelling it in frequency f_d. Since the SINR gain is symmetric with respect to $PSD(f_o)$ and $PSD(f_d)$ as seen in (20), the same improvement in terms of SINR is obtained if the crosstalk component in frequency f_o is estimated first and the interference in frequency f_d is cancelled next.

The data rate increase ΔR is

$$\begin{aligned} \Delta R &= W \log_2 \left(\frac{1 + \frac{SINR \cdot Gain}{\Gamma}}{1 + \frac{SINR}{\Gamma}} \right) \\ &= W \log_2 \left(1 + \frac{(Gain - 1)}{\frac{\Gamma}{SINR} + 1} \right) \qquad (21) \end{aligned}$$

where Γ is the gap [21] from the capacity and W is the subchannel bandwidth. The data rate improvement ΔR is a monotonic increase function of SINR. The higher the original SINR is, the larger the data rate increase is. Therefore, the crosstalk should be estimated in the frequency with a lower SINR and cancelled in the frequency with a higher SINR. This is especially important when SINR is small in one frequency and is large in its dual frequency. ■

The following example illustrates two important special cases for the SINR gain.

Example 2: (a) The interference levels are the same in the dual frequencies and more than 10dB larger than noise, then the SINR gain is approximately

$$Gain \approx \frac{PSD(f_o)}{2\sigma_n^2},$$

which is equal to the interference to noise ratio minus $3dB$.

(b) The interference levels have large disparity, for example, $||c(f_o)||^2 > 10||c(f_d)||^2$, then the SINR gain is approximately

$$Gain \approx \frac{PSD(f_d)}{\sigma_n^2}$$

which is essentially equal to the interference to noise ratio of the smaller crosstalker.

In the xDSL systems, the crosstalk signals from HDSL or SDSL have a large percentage of excess bandwidth. Therefore, the crosstalk signal can be estimated first in the excess band and then be used to cancel the interference in the main lobe. The crosstalk in the excess band is generally smaller than those in the main lobe, which fits case (b) in the above example. The SINR gain in the main lobe is equal to the interference to noise ratio in the excess band. If there is no excess bandwidth, then there is no SINR gain at all!

B.2 Decision-based Approach

For a given pair of the dual frequencies (f_o and $1/T_c - f_o$), the best SINR gain by the MMSE approach above is equal to the smaller one of the interference to noise ratios in these two frequencies. The decision-based approach could do better if the crosstalk signal can be reliably detected. For example, if the crosstalk can be detected reliably in the excess band, then all the interference in the main lobe can be eliminated, as opposed to the residual interference (dotted line) shown in Fig. 2. In this approach, the best choices of the frequency bins for set S_1 are those with large enough interference to noise ratio for the reliable detection of the crosstalk signal. If there are no such choices, the MMSE approach should be used instead.

C. Fast Computation

The crosstalk channel response is changing block over block because the crosstalk signal and the primary signal have different sampling rates. Direct computation of the linear MMSE estimation in (10) requires matrix inversion in every block. A fast computation method is developed to avoid the large matrix inversion, with a slightly degraded performance. This method is based on the well-known fact that the delay in the time domain is equivalent to the phase shift in the frequency domain.

Denote C_0 as the crosstalk function matrix with zero delay ($\tau = 0$), then

$$Q_1 C \approx D_{\tau,1} Q_1 C_0 \tag{22}$$

where $D_{\tau,1}$ represents the phase shift components of the channel response,

$$D_{\tau,1} = \begin{bmatrix} e^{j2\pi \frac{k_1}{N}\frac{\tau}{T}} & 0 & \cdots & 0 \\ 0 & e^{j2\pi \frac{k_2}{N}\frac{\tau}{T}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & e^{j2\pi \frac{k_m}{N}\frac{\tau}{T}} \end{bmatrix}.$$

where $k_1, \cdots, k_m$ are the tone indices in set S_1. The approximation in (22) is because of the edge effect of the matrix. Asymptotically as $M \to \infty$, the approximation becomes an exact equation. The fractional delay τ is easy to infer block over block because the sampling rates of the primary and crosstalk signals are both known.

With this approximation, the estimated signal can be simplified as

$$\tilde{\mathbf{b}} \approx \Psi D^*_{\tau,1} \tilde{\mathbf{z}}_1 \tag{23}$$

where Ψ is a constant matrix

$$\Psi = (C_0^* Q_1^* Q_1 C_0 + \sigma_n^2 I)^{-1} C_0^* Q_1^* \tag{24}$$

which can be pre-computed and stored. This constant matrix avoids matrix inversion and large matrix multiplications in each transmission block. The multiplication $D^*_{\tau,1}\tilde{\mathbf{z}}_1$ implies that the receiver adjusts the timing offset of the signal $\tilde{\mathbf{z}}_1$. Multiplying the constant matrix Ψ roughly represents that the adjusted signal is passed through a linear MMSE filter. The reconstructed signal in (11) can also be simplified as

$$X_c \approx D_{\tau,2} \Phi D^*_{\tau,1} \tilde{\mathbf{z}}_1 \tag{25}$$

where Φ is another constant matrix

$$\Phi = Q_2 C_0 (C_0^* Q_1^* Q_1 C_0 + \sigma_n^2 I)^{-1} C_0^* Q_1^* \tag{26}$$

and $D_{\tau,2}$ is similar to $D_{\tau,1}$ except that it uses the tone indices of set S_2 in the diagonal, *e.g.*, $e^{j2\pi\frac{l}{N}\frac{\tau}{T}}, l \in S_2$. Similarly, the computational complexity is reduced dramatically because the constant matrix Φ can be pre-computed and stored.

D. Extension to Multiple Crosstalkers

The idea of estimating the crosstalk signal in one set of frequency bands and cancelling it in another set of frequency bands can be extended to more than one crosstalker. The system model remains the same as (8), but the crosstalk component $QC\mathbf{b}$ is modified slightly to include more crosstalkers, i.e.,

$$C = [C_1,\ C_2,\ \cdots, C_k]$$
$$\mathbf{b} = [\mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_k]^{\mathrm{T}}$$

where k is the number of the crosstalkers. The amount of performance improvement depends on the crosstalkers' characteristics and the choice of the cancellation approaches. The following example illustrates one particular approach of successive cancellation of the crosstalk signals by estimating the crosstalk signals in the excess band.

Example 3: The crosstalkers HDSL and SDSL have single-sided bandwidth of $192kHz$ and $520kHz$. Both of them have about 100% excess bandwidth. Therefore, SDSL can be estimated first in frequency band $[520 - 1040kHz]$ where the HDSL crosstalker signal does not exist. Then the SDSL crosstalker can be cancelled in the main lobe $[138 - 520kHz]$. After that, the same process is used to estimate and cancel the smaller-bandwidth HDSL crosstalk.

IV. Simulation Results

The ADSL downstream transmission is more vulnerable to NEXT, because the primary signal attenuates very rapidly while NEXT increases as frequency increases. The simulations are thus concentrated on the downstream receiver on the customer side. In the current deployment, the strong NEXT mainly comes from ISDN, HDSL, SDSL, T1 or their repeaters. For a given line, there is very likely only one dominant crosstalk because of the following two reasons. First, the crosstalk line should reside physically close to the victim line. Second, most services in the same bundle are deployed with ADSL. With the

xDSL	ADSL	HDSL
Line code	DMT	2B1Q
Sampling rate f_o $(ks/\sec)$	2208	392
Power (dBm)	19.0	13.6
Duplexing	$up: 26 - 138kHz$ $down: 138 - 1104kHz$	Dual

TABLE I

Main ADSL and HDSL Characteristics.

above two justifications, the simulations assume only one NEXT for a given line.

The main characteristics of ADSL and HDSL are summarized in Table I. For more information, refer to [1][3] and the references therein. SDSL [22] has the same characteristic as HDSL except that it offers variable symmetric data rate. The single-sided PSD of SDSL and HDSL is

$$PSD(f) = K \cdot \frac{2}{f_{sym}} \sin c^2 \left(\frac{\pi f}{f_{sym}} \right) \frac{1}{1 + \left(\frac{f}{\frac{240}{392} f_{sym}} \right)^8} \tag{27}$$

Watts/Hz, where f_{sym} is the symbol rate and $K = \frac{5}{9}\frac{2.7^2}{135}$. The PSD includes a 4th order lowpass butterworth filter whose $3dB$ attenuation occurs at frequency $240 f_{sym}/392$. For HDSL, $f_{sym} = 392kHz$ and $f_{3dB} = 240kHz$ (see [2]). The NEXT coupling functions are taken from the real measured data. Fig. 3 shows several dominant NEXT coupling functions for a given line. The thickest line in the figure is used here for simulation. The crosstalk transfer function is then a cascade response of the rectangular pulse, the butterworth filter, the NEXT coupling function, and the receiver lowpass filter. A linear phase is assumed in the NEXT coupling function.

The system parameters used to calculate the ADSL downstream data rate are summarized in Table II. The upstream and downstream bands for ADSL are shown in Table I. The total noise for the data rate computation is the sum of the background noise, 24

[2]In the original ADSL test procedures as specified in ITU-T G.996.1 Recommendations, $f_{3dB} = 192$kHz. This number has been changed in [22] to 240kHz.

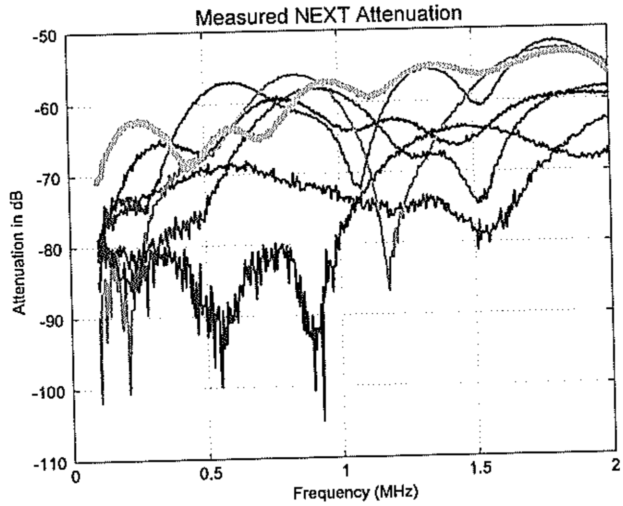

Fig. 3. **Measured NEXT attenuation for a given line.**

ADSL line type:	26 Gauge
AWGN (single-sided):	$-140dBm/Hz$
System margin:	$6dB$
Coding gain:	$3dB$
SNR-gap:	$\Gamma = 9.8dB$
# of self-FEXTs:	24

TABLE II

ADSL system parameters for data rate calculation.

FEXTs from ADSL services by the standard model [12], and one dominant NEXT from HDSL or SDSL.

A. HDSL NEXT Cancellation

HDSL has a relatively small bandwidth with respect to the ADSL signal. The crosstalk is estimated and cancelled by the proposed approaches in Section III-A. Fig. 4 shows three PSDs which correspond to the original NEXT, the residual NEXT using the decision-aided cancellation approach, and the residual NEXT using the MMSE estimation and cancellation approach. The crosstalk signal is observed in the HDSL excess band [198 –

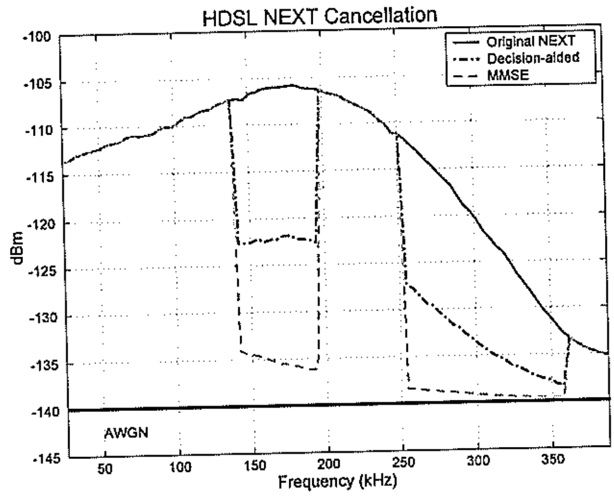

Fig. 4. **HDSL NEXT estimation and cancellation.**

$250kHz$] and cancelled in the main band [$142 - 194kHz$]. In the observing band, the interference is treated as a Gaussian noise for the detection of the primary signal and no improvement has been achieved. However, in the main lobe [$142 - 194kHz$], the improvement by the MMSE estimation and cancellation is very large as shown in Fig. 4, which is about +30 dB. This result fits very well with the analysis and geometric interpretation of the crosstalk cancellation in Section III-B. As expected in Example 2, the SINR gain in frequency $194kHz$ is approximately equal to the interference to noise ratio in the dual frequency $198kHz$ minus $3dB$. If self-NEXTs from other ADSL services in the upstream band [$26-138kHz$] are small (this figure assumes no self-NEXTs), the HDSL crosstalk signal can also be estimated in the ADSL upstream band and then cancelled in the dual HDSL excess band [$254-366kHz$]. The interference in the excess band [$254-366kHz$] can be almost eliminated because the crosstalk signal in band [$26 - 138kHz$] has a much larger interference to noise ratio.

Interestingly, the MMSE estimation and cancellation technique works much better than the decision-aided technique for the HDSL crosstalker. The direct cause is that the HDSL signal are not detected reliably enough and the wrong decisions double the negative impact on the cancellation residual error. The decoding error is mainly because the HDSL crosstalk signal in the voice band [0-26kHz] is lost after it passes the ADSL receiver filter.

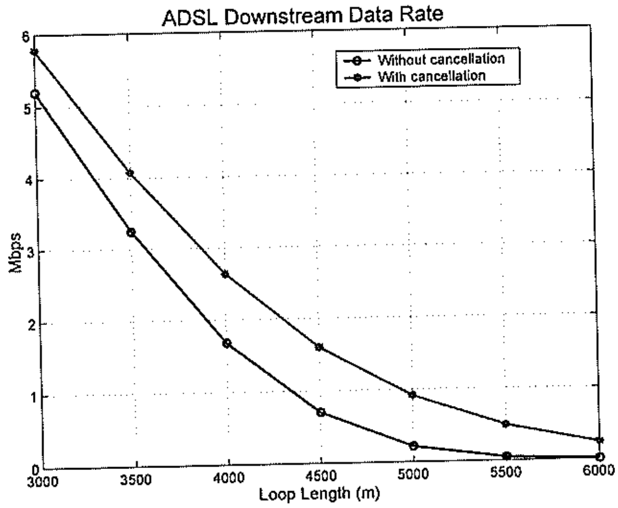

Fig. 5. **Downstream data rates with/without NEXT cancellation.**

The problem is even worse when the HDSL signal can not be estimated in the ADSL upstream band $[26 - 138kHz]$ if self-NEXTs from ADSL are stronger than the HDSL signal.

Fig. 5 shows the ADSL downstream data rate improvement as a result of the MMSE crosstalk estimation and cancellation. Given a line length of $4500m$, the data rate increases from $0.8Mbps$ to $1.7Mbps$. It is also interesting to note that, given the data rate of $0.8Mbps$, the line reach increases from $4500m$ to $5300m$, which is about 39% more coverage area for the service providers.

B. SDSL NEXT Cancellation

SDSL offers variable symmetric data rates by using different bandwidth. In the simulation, the symbol rate is chosen as $f_{sym} = 1040kHz$. Three PSDs of the SDSL NEXT are shown in Fig. 6. They correspond to the original NEXT, the residual NEXT using the decision-aided cancellation approach, and the residual NEXT using the MMSE cancellation approach, respectively. The crosstalk signal is estimated in the SDSL excess band $[522 - 902kHz]$ and cancelled in the dual main lobe $[138 - 518kHz]$. The improvement is very large in those frequency band around $f_{sym}/2$ where the excess band has very high interference to noise ratio. As the frequency increases towards f_{sym}, the crosstalk signal

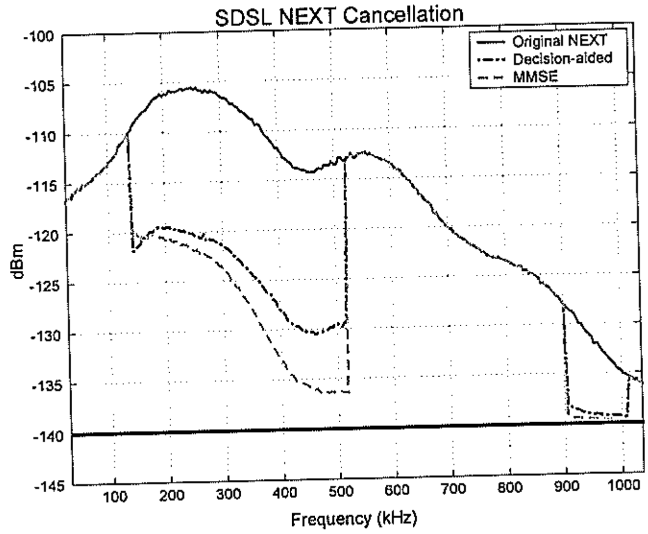

Fig. 6. **SDSL NEXT estimation and cancellation.**

becomes small, correspondingly the improvement of the cancellation in the dual frequency band becomes small. As shown in Fig. 6, the improvement of SINR is smaller in the bands closer to the minimum downstream frequency ($138kHz$). If the self-NEXTs from other ADSL services are small, the crosstalk signal can be estimated in the ADSL upstream band $[26 - 138kHz]$ and cancelled in the dual excess band $[904 - 1014kHz]$.

The MMSE estimation and cancellation approach performs better than the decision-aided approach. However, the difference from these two approaches is smaller than that of the HDSL service. The reason is that the SDSL service has a much larger bandwidth than the HDSL service and the loss of the signal in the low-frequency band $[0 - 26kHz]$ has a relatively smaller effect on the decoding error.

The ADSL data rate improvement is shown in Fig. 7. The improvement is most significant in the loops around $2500 - 4500m$. The improvement is smaller in the shorter loop ($< 2500m$) because the self-FEXTs are larger. Given a line length of $3500m$, the data rate increases from $0.9Mbps$ to $2.7Mbps$. Similarly, for a given data rate of $0.9Mbps$, the loop length increases from $3500m$ to $4600m$, which is about 73% more area coverage for a service provider. The SDSL NEXT is a very severe crosstalk to the ADSL downstream transmission because it has a large bandwidth. The cancellation method proposed in this paper can greatly improve the system performance if there is one (dominant) NEXT.

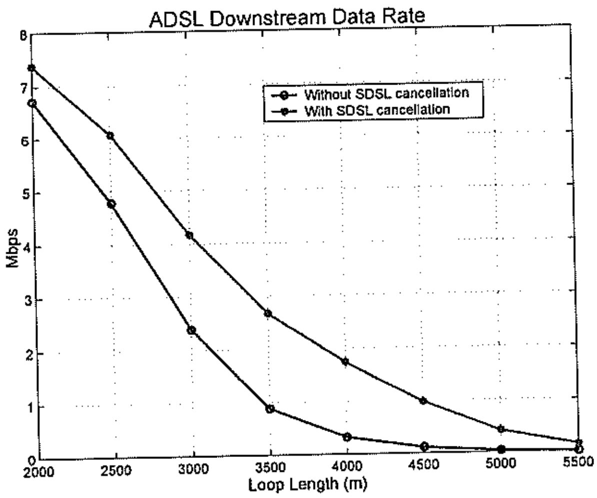

Fig. 7. **Downstream data rates with/without SDSL NEXT cancellation.**

C. Fast Computation

One important aspect of the proposed techniques is that there exists a fast computational scheme, which may be used for practical implementation. This fast algorithm degrades the system performance because of the edge effect in the crosstalk function matrix as described in Sec. III-C. Figs. 8 and 9 show the NEXT cancellation results for HDSL and SDSL, respectively. The three PSDs correspond to the original NEXT, the residual NEXT by the fast algorithm, and the residual NEXT by the accurate MMSE estimation and cancellation in Sec. III-A. The simulation results show that the SINR loss due to the fast algorithm is relatively small.

V. Conclusions

The crosstalk from other types of services, such as HDSL, SDSL, and T1, limits the data rate or the line reach of an ADSL service. This paper presents two new methods to mitigate such NEXT. Both methods are based on the idea of estimating the crosstalk signal in some frequency bands and canceling it in other frequency bands.

The decision-aided method decodes the crosstalk signal based on the observation in some frequency bands. The crosstalk signal is then reconstructed and cancelled in other bands. The method has the problem of error propagation. The MMSE estimation and

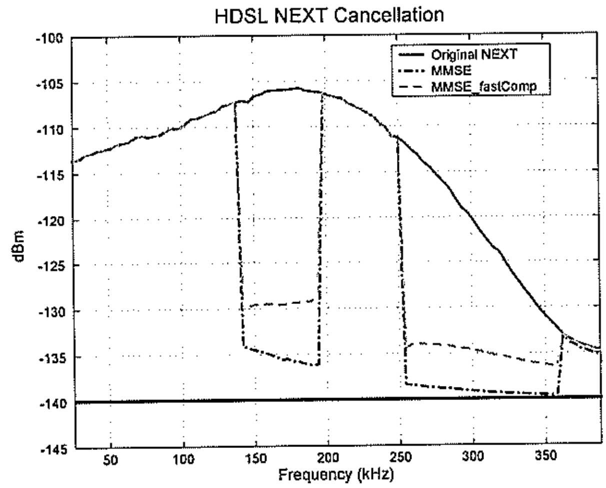

Fig. 8. **HDSL NEXT cancellation using the fast algorithm.**

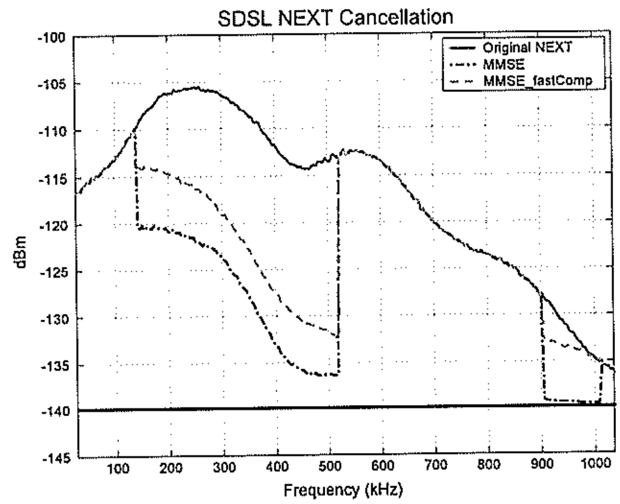

Fig. 9. **SDSL NEXT cancellation using the fast algorithm.**

cancellation method does not decode the crosstalk signal, but uses the MMSE estimation result directly to construct the interference and cancel it in other frequency bands. The latter approach is particularly better to suppress one HDSL or SDSL NEXT because the decision-aided approach can not decode the crosstalk signal reliably due to the signal loss in the low-frequency band $[0 - 26kHz]$. This conclusion is verified by the simulation results.

This paper also presents an intuitive explanation of crosstalk cancellation, which provides the guidance to select the right frequency bands to estimate the crosstalk signal. One interesting result of the MMSE estimation and cancellation is that the gain of interference suppression is the same no matter the crosstalk signal is estimated in the main lobe or in the excess band. In general, the main lobe has larger interference than the excess band. Therefore, if the crosstalk signal is estimated in the main lobe, the interference caused by the excess band can be eliminated. Conversely, if the crosstalk signal is estimated in the excess band, the SINR gain in the main lobe is equal to the interference to noise ratio in the excess band. The larger the interference in the excess band, the better the suppression of the interference in the main lobe.

Moreover, a fast computational algorithm is developed for practical implementation with slightly degraded performance. This method avoids matrix inversion and large matrix multiplication in every transmission block. In contrast, the traditional MMSE estimation requires high computational complexity since the crosstalk signal is non-stationary and large matrix inversion is need in every transmission block.

In conclusion, the MMSE estimation and cancellation scheme is very effective in cancelling the crosstalk if the crosstalk has a large percentage of excess bandwidth. In current xDSL systems, NEXT to an ADSL receiver has a large excess bandwidth, this method is very effective to cancel one dominant NEXT.

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Performance of Crosstalk Cancellation in VDSL

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Abstract

Crosstalk in DSL leads to significant performance degradation and large losses in data-rate. Several crosstalk cancellation techniques have been proposed to address this problem, however, in the existing literature the analysis of these approaches is based on SNR calculations and the SNR-gap approximation. Furthermore, for crosstalk cancellation techniques based on decision-feedback, the effect of error propagation is completely ignored. This makes it hard to predict the performance of crosstalk cancellation in real life, and to see if the significant potential gains can actually be realized. To address this problem, this paper uses Monte-Carlo simulation to investigate the performance of the various crosstalk cancellation techniques that have been proposed. The effect of noise-enhancement in zero-forcing crosstalk cancellers and error-propagation in decision-feedback cancellers is examined. The results confirm that a very simple crosstalk cancellation structure can achieve near-optimal performance.

1. Introduction

Modern Digital Subscriber Line (DSL) systems use frequencies up to 12 MHz in transition. This leads to electromagnetic coupling between nearby twisted-pairs within the cable binder, an effect known as *crosstalk.* Crosstalk is a major source of performance degradation and significantly limits the data-rate and reach at which a DSL service may be provided[1].

Crosstalk between modems on the same end of the loop is referred to as near-end crosstalk (NEXT) whilst crosstalk on different ends of the loop is referred to as far-end crosstalk (FEXT). NEXT is typically avoided by using frequency division duplexing (FDD); however FEXT is still a major problem in most DSL systems. This is particularly true when one of the transmitters is located much closer to the receiving modems than all other transmitters. The crosstalk from this transmitter can often be stronger than the signals of interest on the other lines, leading to a total loss of service. This so-called *near-far problem* is particularly evident in upstream VDSL transmission when a customer premises (CP) modem is located further upstream of the other modems in the network.

Crosstalk cancellation has been proposed as one way of addressing the crosstalk problem. This technique is based on the concept of jointly processing the received signals of all lines in order to filter out the crosstalk whilst preserving the signal of interest[2].

In previous literature a decision-feedback crosstalk canceller was shown to achieve near-optimal performance[2]. However this analysis was based on the assumption of error-free detection and hence the effects of error propagation were not accounted for. More recent work showed that a simple linear Zero Forcing (ZF) crosstalk canceller could achieve near-optimal performance[4]. However this work was based on an SNR-gap approximation, which may not accurately reflect real-life performance.

This paper uses Monte-Carlo simulation to investigate the performance, in terms of symbol-error rate, of the different proposed crosstalk cancellers. In particular, we are looking to confirm the conclusions made in previous analytical work, and make a specific study on the effects of noise-enhancement and error-propagation on crosstalk canceller performance.

The remainder of the paper is arranged as follows. Section II gives an overview of the system model. Section III presents the different crosstalk cancellation techniques. Section IV describes the performance, in terms of symbol-error rate, of the different crosstalk cancellers as we vary the disturbing modem's line length. Section V draws conclusions.

2. Channel Model

The channel model considered here is of the form

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k . \qquad (1)$$

Here $\mathbf{x}_k = [x_k^1 \cdots x_k^N]^T$ is the vector of symbols transmitted on tone k, where x_k^n is the symbol transmitted by user n on tone k. Similarly $\mathbf{y}_k = [y_k^1 \cdots y_k^N]^T$ is the vector of symbols received on tone k. The vector $\mathbf{z}_k = [z_k^1 \cdots z_k^N]^T$ is the additive noise experienced by the receivers on tone k and incorporates radio frequency interference (RFI), thermal noise and alien crosstalk. We assume that $\mathbf{z}_k$ is white and Gaussian.

The crosstalk channel matrix is denoted by $\mathbf{H}_k = [h_k^{n,m}]$. The diagonal element $h_k^{n,n}$ is the direct channel from transmitter n to receiver n, whilst the off-diagonal element $h_k^{n,m}$ is the crosstalk channel from transmitter m to receiver n.

In upstream transmission the receiving modems are co-located at a common central office. As a result the diagonal element of any column of the channel matrix $\mathbf{H}_k$ will have a much larger magnitude than the off-diagonal elements of that column, that is

$$\left|h_k^{n,n}\right| >> \left|h_k^{m,n}\right|, \forall m.$$

We refer to this as column-wise diagonal dominance (CWDD)[4].

3. Equalization Techniques

3.1 Single User Equalization

We begin by examining the performance of a conventional DSL modem which does not employ crosstalk cancellation. Each receiver first equalizes the received signal by dividing it by the direct channel gain

$$\hat{x}_k^n = y_k^n / h_k^{n,n} .$$

Since no cancellation is applied, the user will suffer the full effects of crosstalk.

3.2 Zero Forcing Canceller

The zero forcing canceller estimates the transmitted symbols by multiplying the received symbol vector with the inverse of the channel matrix, hence

$$\hat{\mathbf{x}}_k = (\mathbf{H}_k)^{-1} \mathbf{y}_k .$$

The zero-forcing canceller has a linear design, which leads to a low run-time complexity and low-latency. One potential disadvantage of the zero-forcing approach is that it may lead to severe noise-enhancement if the crosstalk channel matrix is poorly conditioned.

Thankfully, in DSL channels CWDD has been shown to ensure a well conditioned channel matrix, leading to near-optimal performance of the zero-forcing canceller[4]. This theoretical result is confirmed through our simulations in Section IV where we see that the zero-forcing canceller achieves near-optimal performance

3.3 Decision Feedback Canceller

Decision feedback equalization is traditionally used for canceling inter-symbol interference, however this approach has also been proposed for crosstalk cancellation in DSL[2].

As seen in figure 1, the decision feedback canceller consists of a feed-forward and a feedback filter. The feed-forward filter converts the crosstalk channel matrix into one that is upper triangular, and hence the crosstalk obeys a form of causality, in the sense that each user only experiences crosstalk from previous users. This allows decision feedback to be used to detect each of the users in turn, before subtracting the crosstalk they cause to the remaining undetected users[2].

In practice this is implemented through a QR decomposition of the crosstalk channel matrix

$$\mathbf{H}_k = \mathbf{Q}_k \mathbf{R}_k .$$

Here $\mathbf{Q}_k$ is a unitary matrix, whilst $\mathbf{R}_k$ is upper triangular.

The matrix $\mathbf{Q}_k^H$ forms the feed-forward filter which transforms the received vector of (1) to

$$\mathbf{w}_k = \mathbf{Q}_k^H \mathbf{y}_k = \mathbf{R}_k \mathbf{x}_k + \mathbf{Q}_k^H \mathbf{z}_k .$$

Since $\mathbf{Q}_k$ is unitary, the feed-forward filter does not alter the statistics of the noise $\mathbf{z}_k$, which we assume to be spatially white. If the noise is spatially coloured then a pre-whitening operation needs to be integrated into the feed-forward filter[5]. This is relatively straight-forward to implement in practice, and has no effect on complexity, so we continue under the assumption of spatially white noise.

Now that the channel has been converted into an upper-triangular matrix $\mathbf{R}_k$, decision feedback can be applied to cancel the remaining crosstalk. The estimate for user n is formed by subtracting the crosstalk components of the previously detected users

$$\hat{x}_k^n = \text{dec}\left[\frac{w_k^n}{r_k^{n,n}} - \sum_{m=n+1}^{N} \frac{r_k^{n,m}}{r_k^{n,n}} \hat{x}_k^m\right],$$

where $w_k^n = [\mathbf{w}_k]_n$ and $r_k^{n,m} = [\mathbf{R}_k]_{n,m}$.

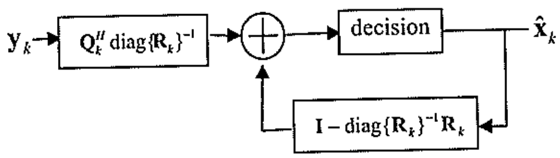

Figure 1: Decision feedback equalizer

The decision feedback canceller will perform very close to the theoretical channel capacity provided that the previously detected users have been detected error-free[2]. In practice this is not the case, and each user will experience errors due to the noise within the channel. When a user is erroneously detected, the decision feedback operation will actually create more crosstalk, leading to error-propagation and a significant reduction in performance.

An important contribution of this work is to examine the effect of decision errors in the decision feedback canceller. This is something that has been ignored in prior work that is based on the assumption of error-free detection, an assumption that is of course invalid in practice[2].

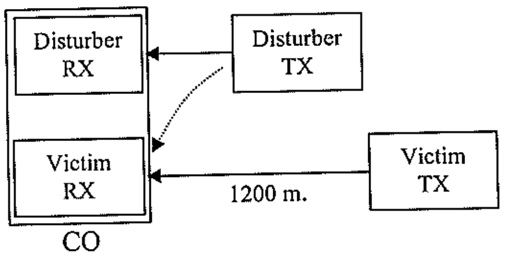

Figure 2: Upstream VDSL simulation environment

4. Performance

The performance of single user detection, zero forcing cancellation and decision feedback cancellation will be compared in this section. The simulation scenario is depicted in Fig. 2. The victim line was fixed at a length of 1200 m. whilst the length of the disturbing line was allowed to vary from 100 m. to 1200 m.

The symbol error rate was calculated on tone 1205, which corresponds to the highest tone in upstream band 1 under the 998 FDD band plan[3]. In simulation we assume a line diameter of 0.5 mm (24 AWG). Empirical models were used to generate the direct and crosstalk channels [3]. The AWGN was assumed to have a PSD of −133 dBm/Hz for all tones and lines. Tone spacing was set to 4.3125 kHz and the DMT symbol-rate was set to 4 kHz as per VDSL standards[3].

The symbol error rate (SER) on the 1200 m. victim line is shown in Fig. 3 as we vary the length of the disturber's line. In the case of single-user detection (no cancellation) the SER is highest for short disturber line lengths. This is to be expected since for short disturber line lengths the near-far effect will be most severe, as the crosstalk signal travels only a short distance into the victim, completely dominating the signal of interest, which has already attenuated quite significantly over the 1200 m. line.

Clearly the application of crosstalk cancellation, either zero-forcing or decision feedback, leads to a significant reduction in SER, and brings the unencoded SER down to an acceptable level of 2×10^{-4}, which corresponds to a coded symbol error rate of 10^{-7}. In practice this will allow many more tones to be used for transmission, increasing the overall data-rate. This corresponds quite nicely with the results seen in previous work, which predicted large data-rate gains from an analytical perspective[2][4]. In this paper we have confirmed these benefits through more accurate Monte-Carlo simulation.

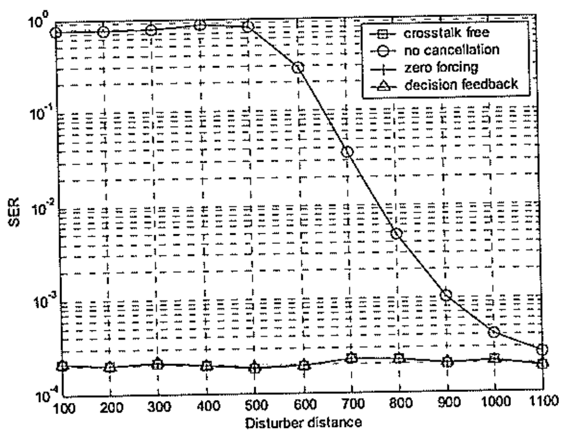

Figure 3: Crosstalk cancellation performance

It is interesting to note that both the zero-forcing and decision feedback cancellers operate very close to the crosstalk free bound, so both of these techniques exhibit near-optimal performance.

This confirms prior analytical work, which showed that the zero-forcing canceller causes negligible noise enhancement due to the CWDD nature of DSL channels.

It also reveals a previously unsuspected insight, that the performance of the decision feedback canceller is relatively unaffected by decision error propagation.

Hence we have confirmed that the zero-forcing and decision feedback cancellers are simple, low complexity designs with near-optimal performance. This is an observation alluded to through analysis in previous work, and in this paper we see the same result, this time reinforced through numerical simulation.

5. Conclusions

This paper compared the performance of the zero forcing and decision feedback cancellers. Unlike previous work, a comparison was made on the basis of numerical simulation rather than through analysis alone. This allowed the effects of noise enhancement and decision error propagation to be evaluated directly, issues that were ignored in the analysis of previous work.

It was seen that both the zero forcing and decision feedback cancellers achieve near optimal performance. The zero-forcing canceller has a lower run-time complexity and is preferable when the noise is spatially white. When strong alien crosstalk sources exist near the central office, the noise will be correlated between lines. CWDD property of the crosstalk channel matrix, and the zero-forcing canceller then loses its near-optimal In this case a noise pre-whitening operation must be performed. This noise pre-whitening often destroys the performance. In this case the decision feedback canceller is preferable. An important area for future work is a more detailed study into the effects of noise correlation on crosstalk canceller performance.

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A Matrix Theoretic Derivation of the Kalman Filter*

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Abstract

We present a matrix theoretic derivation of the Kalman filter—motivated by the statistical technique of minimum variance estimation—in order to make its theoretical underpinnings accessible to a broader audience. Standard derivations of the filter utilize probabilistic arguments that are less familiar to the matrix analyst and computational mathematician.

1 Introduction

Of key interest to us are stochastic linear systems of the form

$$\mathbf{d} = \mathbf{G}\boldsymbol{\phi} + \mathbf{e}, \tag{1}$$

where $\mathbf{d} \in \mathbb{R}^m$ is measured data; $\mathbf{G} \in \mathbb{R}^{m \times n}$ is a known model matrix; $\boldsymbol{\phi} \in \mathbb{R}^n$ is the unknown parameter vector to be estimated; $\mathbf{e} \in \mathbb{R}^m$ is a zero-mean Gaussian random vector; and $\boldsymbol{\phi} \in \mathbb{R}^n$ is a zero-mean Gaussian random vector. In this case, the minimum variance estimator of $\boldsymbol{\phi}$ [4] can be expressed as the solution of a $n \times n$ linear system.

Our derivation of the Kalman filter [3, 5] arises from an application of minimum variation estimation to the sequentially coupled system of linear equations

$$\begin{aligned} \boldsymbol{\phi}_k &= \mathbf{M}_k \boldsymbol{\phi}_{k-1} + \mathbf{E}_k, & (2)\\ \mathbf{d}_k &= \mathbf{G}_k \boldsymbol{\phi}_k + \mathbf{e}_k, & (3) \end{aligned}$$

where equation (3) is analogous to (1); and in (2), $\mathbf{M}_k \in \mathbb{R}^n$ is the known evolution matrix, $\boldsymbol{\phi}_{k-1}$ is a Gaussian random vector, and $\mathbf{E}_k \in \mathbb{R}^n$ is a zero-mean Gaussian random vector.

Our discussion requires a knowledge of some basic statistical definitions and results.

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2 Statistical Preliminaries

Let $\mathbf{x} = (x_1, \ldots, x_n)^T$ be a random vector with $E(x_i)$ the mean of x_i and $E((x_i - \mu_i)^2)$, where $\mu_i = E(x_i)$, its variance. The mean of $\mathbf{x}$ is then defined $E(\mathbf{x}) = (E(x_1), \ldots, E(x_n))^T$, while the $n \times n$ covariance matrix of $\mathbf{x}$ is defined

$$[\text{cov}(\mathbf{x})]_{ij} = E((x_i - \mu_i)(x_j - \mu_j)), \quad 1 \leq i, j \leq n.$$

Note that the diagonal of $\text{cov}(\mathbf{x})$ contains the variances of $x_1, \ldots, x_n$, while the off diagonal elements contain the covariance values. Thus if x_i and x_j are independent $[\text{cov}(\mathbf{x})]_{ij} = 0$.

The $n \times m$ cross correlation matrix of the random n-vector $\mathbf{x}$ and m-vector $\mathbf{y}$, which we will denote $\mathbf{\Gamma_{xy}}$, is defined

$$\mathbf{\Gamma_{xy}} = E(\mathbf{x}\mathbf{y}^T), \tag{4}$$

where $[E(\mathbf{x}\mathbf{y}^T)]_{ij} = E(x_i y_j)$. If $\mathbf{x}$ and $\mathbf{y}$ are independent, then $\mathbf{\Gamma_{xy}}$ is the zero matrix. Furthermore,

$$E(\mathbf{x}) = 0 \quad \text{implies} \quad \mathbf{\Gamma_{xx}} = \text{cov}(\mathbf{x}). \tag{5}$$

Finally, given an $m \times n$ matrix $\mathbf{A}$ and a random n-vector $\mathbf{x}$, it is not difficult to show that

$$\text{cov}(\mathbf{A}\mathbf{x}) = \mathbf{A}\text{cov}(\mathbf{x})\mathbf{A}^T. \tag{6}$$

We end these preliminary comments with the example of primary interest to us in this paper, the Gaussian distribution. If $\mathbf{d}$ is an $n \times 1$ Gaussian random vector, then its probability density function has the form

$$p_{\mathbf{d}}(\mathbf{d}; \boldsymbol{\mu}, \mathbf{C}) = \frac{1}{\sqrt{(2\pi)^n \det(\mathbf{C})}} \exp\left(-\frac{1}{2}(\mathbf{d} - \boldsymbol{\mu})^T \mathbf{C}^{-1}(\mathbf{d} - \boldsymbol{\mu})\right), \tag{7}$$

where $\boldsymbol{\mu} \in \mathbb{R}^n$, $\mathbf{C}$ is an $n \times n$ symmetric positive definite matrix, and $\det(\cdot)$ denotes matrix determinant. Then $E(\mathbf{d}) = \boldsymbol{\mu}$ and $\text{cov}(\mathbf{d}) = \mathbf{C}$. We will use the notation $\mathbf{d} \sim N(\boldsymbol{\mu}, \mathbf{C})$ in this case.

For more details on introductory mathematical statistics, see one of many introductory mathematics statistics texts, including [2].

3 Minimum Variance Estimation

We consider linear system (1) and assume that $\phi \sim N(0, C_\phi)$ with C_ϕ a symmetric positive definite matrix. We assume, furthermore, that ϕ and $\mathbf{e}$ are independent random variables. The problem of estimating ϕ is statistical, however as we will see, the minimum variance estimator can be expressed as the solution of a linear system. We can now define the minimum variance estimator of ϕ.

Definition 1. *Suppose ϕ and $\mathbf{d}$ are jointly distributed random vectors whose components have finite expected squares. The* minimum variance estimator *of ϕ from $\mathbf{d}$ is given by*

$$\phi^{est} = \hat{\mathbf{B}}\mathbf{d},$$

where

$$\hat{\mathbf{B}} = \arg \min_{\mathbf{B}\in\mathbb{R}^{n\times n}} E\left(\|\mathbf{Bd} - \phi\|\|_2^2\right).$$

We now state and prove a theorem that gives us the minimum variance estimator in an elegant closed form.

Theorem 1. *If $\Gamma_{\mathbf{dd}}$ (defined in (4)) is invertible, then the minimum variance estimator of ϕ from* $\mathbf{d}$ *is given by*

$$\phi^{est} = (\Gamma_{\phi\mathbf{d}}\Gamma_{\mathbf{dd}}{}^{-1})\mathbf{d}.$$

Proof. First we note, via properties of the trace function, that

$$\begin{aligned} E(\|\mathbf{Bd} - \phi\|^2) &= \text{trace}(E[(\mathbf{Bd} - \phi)(\mathbf{Bd} - \phi)^T]), \\ &= \text{trace}\left(\mathbf{B}E[\mathbf{dd}^T]\mathbf{B}^T - \mathbf{B}E[\mathbf{d}\phi^T] - E[\phi\mathbf{d}^T]\mathbf{B}^T + E[\phi\phi^T]\right). \end{aligned}$$

Then, using the distributive property of the trace function and the identity

$$\frac{d}{d\mathbf{B}}\text{trace}(\mathbf{BC}) = \frac{d}{d\mathbf{B}}\text{trace}(\mathbf{C}^T\mathbf{B}^T) = \mathbf{C}^T,$$

we see that $dE(\|\mathbf{Bd} - \phi\|^2)/d\mathbf{B} = \mathbf{0}$ when

$$\mathbf{B} = \Gamma_{\phi\mathbf{d}}\Gamma_{\mathbf{dd}}{}^{-1},$$

where $\Gamma_{\phi\mathbf{d}}$ and $\Gamma_{\mathbf{dd}}$ are defined in (4). □

Given our assumptions above – recall that ϕ and $\mathbf{e}$ are independent zero mean random vectors – we have from (4) and (5)

$$\begin{aligned} \Gamma_{\phi\mathbf{d}} &= \Gamma_{\phi\phi}\mathbf{G}^T + \Gamma_{\phi\mathbf{e}} = C_\phi\mathbf{G}^T, \\ \Gamma_{\mathbf{dd}} &= \mathbf{G}\Gamma_{\phi\phi}\mathbf{G}^T + \Gamma_{\mathbf{ee}} = \mathbf{G}C_\phi\mathbf{G}^T + \mathbf{C}_\mathbf{e}. \end{aligned}$$

Thus the minimum variance estimator is given by

$$\begin{aligned} \phi^{est} &= C_\phi\mathbf{G}^T(\mathbf{G}C_\phi\mathbf{G}^T + \mathbf{C}_\mathbf{e})^{-1}\mathbf{d}, \\ &= (\mathbf{G}^T\mathbf{C}_\mathbf{e}^{-1}\mathbf{G} + C_\phi{}^{-1})^{-1}\mathbf{G}^T\mathbf{C}_\mathbf{e}^{-1}\mathbf{d}. \end{aligned} \tag{8}$$

The second equality is valid since C_ϕ is assumed to be nonsingular (a nice exercise), and it shows us that in this case ϕ^{est} can also be expressed

$$\phi^{est} = \arg\min_{\phi} \|\mathbf{G}\phi - \mathbf{d}\|^2_{\mathbf{C}_\mathbf{e}^{-1}} + \|\phi\|^2_{\mathbf{C}_\phi^{-1}}. \tag{9}$$

This establishes a clear connection between minimum variance estimation and generalized Tikhonov regularization [4]. Note in particular that if $\mathbf{C}_\mathbf{e} = \sigma_1^2\mathbf{I}$ and $C_\phi = \sigma_2^2\mathbf{I}$, the minimum variance estimate can be written

$$\phi^{est} = \arg\min_{\phi} \|\mathbf{G}\phi - \mathbf{d}\|_2^2 + (\sigma_1^2/\sigma_2^2)\|\phi\|_2^2,$$

which has classical Tikhonov form.

4 The Kalman Filter

Up to this point, we have considered stationary linear models, but suppose that what we wish to estimate (ϕ above) changes in time. More specifically, suppose our model now has the form (1), (2). Equation (1) is the equation of evolution for ϕ_k with $\mathbf{M}_k$ the $n \times n$ linear evolution matrix, and $\mathbf{E}_k \sim N(0, \mathbf{C}_{\mathbf{E}_k})$. In equation (2), $\mathbf{d}_k$ denotes the $m \times 1$ observed data, $\mathbf{G}_k$ the $m \times n$ linear observation matrix, and $\mathbf{e}_k \sim N(0, \mathbf{C}_{\mathbf{e}_k})$. In both equations, k denotes the time index.

The problem is to estimate ϕ_k at time k from $\mathbf{d}_k$ and an estimate ϕ_{k-1}^{est} of the state at time $k-1$. We assume $\phi_{k-1}^{est} \sim N(\phi_{k-1}, \mathbf{C}_{k-1}^{est})$.

To facilitate a more straightforward application of the result of Theorem 1, we rewrite (1), (2). First, define

$$
\begin{aligned}
\phi_k^a &= \mathbf{M}_k \phi_{k-1}^{est} && (10)\\
\mathbf{z}_k &= \phi_k - \phi_k^a, && (11)\\
\mathbf{r}_k &= \mathbf{d}_k - \mathbf{G}_k \phi_k^a. && (12)
\end{aligned}
$$

Then, subtracting (10) from (1) and $\mathbf{G}_k \phi_k^a$ from both sides of (2), and dropping the k dependence for notational simplicity, we obtain the stochastic linear equations

$$
\begin{aligned}
\mathbf{z} &= \mathbf{M}(\phi - \phi^{est}) + \mathbf{E}, && (13)\\
\mathbf{r} &= \mathbf{G}\mathbf{z} + \mathbf{e}. && (14)
\end{aligned}
$$

The minimum variance estimator of $\mathbf{z}$ from $\mathbf{r}$ given (13), (14) is then given, via Theorem 1, by

$$\mathbf{z}^{est} = \Gamma_{\mathbf{zr}} {\Gamma_{\mathbf{rr}}}^{-1} \mathbf{r}.$$

We assume that $\phi - \phi^{est}$ is independent of $\mathbf{E}$, and that $\mathbf{z} = \phi - \phi^a$ is independent of $\mathbf{e}$. We note, furthermore, that by assumption both of these random vectors have zero mean. Then, from (4), (5), (6), (13) and (14), we obtain

$$
\begin{aligned}
\Gamma_{\mathbf{zz}} &= \mathbf{M}\mathbf{C}^{est}\mathbf{M}^T + \mathbf{C}_{\mathbf{E}} \stackrel{\text{def}}{=} \mathbf{C}^a, && (15)\\
\Gamma_{\mathbf{zr}} &= \mathbf{C}^a \mathbf{G}^T,\\
\Gamma_{\mathbf{rr}} &= \mathbf{G}\mathbf{C}^a\mathbf{G}^T + \mathbf{C}_{\mathbf{e}}.
\end{aligned}
$$

where $\mathbf{C}^{est}$ and $\mathbf{C}^a$ are the covariance matrices for ϕ^{est} and ϕ^a respectively. Thus, finally, the minimum variance estimator of $\mathbf{z}$ is given by

$$\mathbf{z}^{est} = \mathbf{C}^a\mathbf{G}^T(\mathbf{G}\mathbf{C}^a\mathbf{G}^T + \mathbf{C}_{\mathbf{e}})^{-1}\mathbf{r}, \qquad (16)$$

From (16) and (11) we then immediately obtain the Kalman Filter estimate of ϕ given by

$$\phi_+^{est} = \phi^a + \mathbf{H}(\mathbf{d} - \mathbf{G}\phi^a). \qquad (17)$$

where

$$\mathbf{H} = \mathbf{C}^a\mathbf{G}^T(\mathbf{G}\mathbf{C}^a\mathbf{G}^T + \mathbf{C}_{\mathbf{e}})^{-1} \qquad (18)$$

is known as the Kalman Gain.

Finally, in order to compute the covariance of ϕ_+^{est}, we note that by (17) and (2),

$$\phi_+^{est} = (\mathbf{I} - \mathbf{H}\mathbf{G})\phi^a + \mathbf{H}\mathbf{e} + \mathbf{H}\mathbf{G}\phi,$$

where ϕ is the true state. Given our assumptions and using (6), the covariance then takes the form

$$\mathbf{C}_+^{est} = (\mathbf{I} - \mathbf{H}\mathbf{G})\mathbf{C}^a(\mathbf{I} - \mathbf{H}\mathbf{G})^T + \mathbf{H}\mathbf{C}_e\mathbf{H}^T,$$

which can be rewritten, using the identity $\mathbf{H}\mathbf{C}_e\mathbf{H}^T = (\mathbf{I} - \mathbf{H}\mathbf{G})\mathbf{C}^a\mathbf{G}^T\mathbf{H}^T$, in the simplified form

$$\mathbf{C}_+^{est} = \mathbf{C}^a - \mathbf{H}\mathbf{G}\mathbf{C}^a. \tag{19}$$

Incorporating the k dependence again leads directly to the Kalman filter iteration.

The Kalman Filter Algorithm

Step 0: Select initial guess ϕ_0^{est} and covariance $\mathbf{C}_0^{est}$, and set $k = 1$.

Step 1: Compute the evolution model estimate and covariance:
A. Compute $\phi_k^a = \mathbf{M}_k\phi_{k-1}^{est}$;
B. Compute $\mathbf{C}_k^a = \mathbf{M}_k\mathbf{C}_{k-1}^{est}\mathbf{M}_k^T + \mathbf{C}_{\mathbf{E}_k} := \mathbf{C}_k^a$.

Step 2: Compute the Kalman filter estimate and covariance:
A. Compute the Kalman Gain $\mathbf{H}_k = \mathbf{C}_k^a\mathbf{G}_k^T(\mathbf{G}_k\mathbf{C}_k^a\mathbf{G}_k^T + \mathbf{C}_e)^{-1}$;
B. Compute the estimate $\phi_k^{est} = \phi_k^a + \mathbf{H}_k(\mathbf{d}_k - \mathbf{G}_k\phi_k^a)$;
C. Compute the estimate covariance $\mathbf{C}_k^{est} = \mathbf{C}_k^a - \mathbf{H}_k\mathbf{G}_k\mathbf{C}_k^a$.

Step 3: Update $k := k + 1$ and return to Step 1.

4.1 A Variational Formulation of the Kalman Filter

As in the stationary case (see (8)), we can rewrite equation (16) in the form

$$\mathbf{z}^{est} = (\mathbf{G}\mathbf{C}_e^{-1}\mathbf{G}^T + (\mathbf{C}_a)^{-1})^{-1}\mathbf{G}^T\mathbf{C}_e^{-1}\mathbf{r},$$

which, yields, using (11), the Kalman filter estimate

$$\begin{aligned} \phi_+^{est} &= \phi^a + [\mathbf{G}^T\mathbf{C}_e^{-1}\mathbf{G} + (\mathbf{C}^a)^{-1}]^{-1}\mathbf{G}^T\mathbf{C}_e^{-1}(\mathbf{d} - \mathbf{G}\phi^a), \\ &= \arg\min_{\phi}\left\{\ell(\phi) \stackrel{\text{def}}{=} \frac{1}{2}(\mathbf{d} - \mathbf{G}\phi)^T\mathbf{C}_e^{-1}(\mathbf{d} - \mathbf{G}\phi) + \frac{1}{2}(\phi - \phi^a)^T(\mathbf{C}^a)^{-1}(\phi - \phi^a)\right\}. \end{aligned}$$

It can be shown using a Taylor series argument that

$$\phi_+^{est} = \phi^a - \nabla^2\ell(\phi^a)^{-1}\nabla\ell(\phi^a), \tag{20}$$

where $\nabla\ell$ and $\nabla^2\ell$ denote the gradient and Hessian of ℓ respectively, and are given by

$$\begin{aligned} \nabla\ell(\phi) &= \mathbf{G}^T\mathbf{C}_e^{-1}(\mathbf{d} - \mathbf{G}\phi) + (\mathbf{C}^a)^{-1}(\phi - \phi^a), \\ \nabla^2\ell(\phi) &= \mathbf{G}^T\mathbf{C}_e^{-1}\mathbf{G} + (\mathbf{C}^a)^{-1}. \end{aligned}$$

By the matrix inversion lemma, we have

$$(\mathbf{G}^T\mathbf{C}_e^{-1}\mathbf{G} + (\mathbf{C}^a)^{-1})^{-1} = \mathbf{C}^a - \mathbf{C}^a\mathbf{G}^T(\mathbf{G}^T\mathbf{C}^a\mathbf{G}^T + \mathbf{C}_e)^{-1}\mathbf{G}\mathbf{C}^a.$$

Then from equations (18) and (19), we obtain the very useful fact that

$$\mathbf{C}_+^{est} = \nabla^2\ell(\phi)^{-1}. \tag{21}$$

This allows us to define an iteration analogous to that given above.

The Variational Kalman Filter Algorithm

Step 0: Select initial guess ϕ_0^{est} and covariance $\mathbf{C}_0^{est}$, and set $k = 1$.

Step 1: Compute the evolution model estimate and covariance:
A. Compute $\phi_k^a = \mathbf{M}_k\phi_{k-1}^{est}$;
B. Compute $\mathbf{C}_k^a = \mathbf{M}_k\mathbf{C}_k^{est}\mathbf{M}_k^T + \mathbf{C}_{\mathbf{E}_k} := \mathbf{C}_k^a$.

Step 2: Compute the Kalman filter estimate and covariance:
A. Compute the estimate $\phi_k^{est} = \arg\min_{\phi} \ell(\phi)$;
C. Compute the estimate covariance $\mathbf{C}_k^{est} = \nabla^2\ell(\phi)^{-1}$.

Step 3: Update $k := k + 1$ and return to Step 1.

A natural question is, what is the use of this equivalent formulation of the Kalman filter? Theoretically there is no benefit gained in using the variational Kalman filter if the estimate and its covariance are computed exactly. However, with the variational approach, the filter estimate, and even its covariance [1], can be computed approximately using an iterative minimization method. This is particularly important for large-scale problems where the exact Kalman filter is prohibitively expensive to compute.

4.2 The Extended Kalman Filter

The extended Kalman filter (EKF) is the extension of the Kalman filter when (1), (2) are replaced by

$$\begin{aligned} \phi_k &= \mathcal{M}(\phi_{k-1}) + \mathbf{E}_k, & (22)\\ \mathbf{d}_k &= \mathcal{G}(\phi_k) + \mathbf{e}_k, & (23) \end{aligned}$$

where $\mathcal{M}$ and $\mathcal{G}$ are (possibly) nonlinear functions. EKF is obtained by the following simple modification of either of the above algorithms: in Step 1, A use, instead, $\phi_k^a = \mathcal{M}(\phi_k^{est})$, and define

$$\mathbf{M}_k = \frac{\partial\mathcal{M}(\phi_{k-1}^{est})}{\partial\phi}, \quad \text{and} \quad \mathbf{G}_k = \frac{\partial\mathcal{G}(\phi_k^a)}{\partial\phi}, \tag{24}$$

where $\frac{\partial}{\partial\phi}$ denotes the Jacobian.

5 Conclusions

We have presented a derivation of the Kalman filter that utilizes matrix analysis techniques as well as the Bayesian statistical approach of minimum variance estimation. In addition, we presented an equivalent variational formulation of the Kalman filter, as well as the extended Kalman filter for nonlinear problems.

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