Facultad de
Ciencias naturales y matemáticas

INGENIERÍA EN LOGÍSTICA Y TRANSPORTE

AÑO:	2022	**TÉRMINO:**	PRIMERO
MATERIA:	SIMULACIÓN MATEMÁTICA	**PROFESORES:**	DAVID DE SANTIS
EXAMEN:	PRIMERO	**FECHA:**	08-07-2022

COMPROMISO DE HONOR

Yo declaro que he sido informado y conozco las normas disciplinarias que rigen a la ESPOL, en particular el Código de Ética y el Reglamento de Disciplina. Al aceptar este compromiso de honor, reconozco y estoy consciente de que la presente evaluación está diseñada para ser resuelta de forma individual; que puedo comunicarme únicamente con la persona responsable de la recepción de la evaluación; y que al realizar esta evaluación no navegaré en otras páginas que no sean las páginas de Aula Virtual/plataforma de la evaluación; que no recibiré ayuda ni presencial ni virtual; que no haré consultas en libros, notas, ni apuntes adicionales u otras fuentes indebidas o no autorizadas por el evaluador; ni usaré otros dispositivos electrónicos o de comunicación no autorizados. Además, me comprometo a mantener encendida la cámara durante todo el tiempo de ejecución de la evaluación, y en caso de que el profesor lo requiera, tomar una foto de las páginas en las que he escrito el desarrollo de los temas y subirla a Aula Virtual/plataforma de la evaluación, como evidencia del trabajo realizado, estando consciente que el no subirla, anulará mi evaluación. Acepto el presente compromiso, como constancia de haber leído y aceptado la declaración anterior y me comprometo a seguir fielmente las instrucciones que se indican para la realización de la presente evaluación (incluyendo los requisitos de uso de la tecnología). Estoy consciente que el incumplimiento del presente compromiso anulará automáticamente mi evaluación y podría ser objeto del inicio de un proceso disciplinario.

( ) Acepto ( ) No Acepto

Question1

1. Consider a machine with three components which are subject to failures.

- **Component A:** The lifetime of this component is distributed exponentially with rate 0.3/day. The price of this component is affected by the market conditions and hence cost of each component is uniformly distributed between £5 and £10 . Once this component fails, the machine fails as well.
- **Component B:** The lifetime of this component is distributed uniformly between 2 and 4 days. Each component costs £15. Once this component fails, the machine fails as well.
- **Component C:** The lifetime of this component is distributed exponentially with rate 0.6/day. Each component costs £5. This component is not vital for the operation of the machine and the machine continues to run even if this component is in a failed state (as long as both components A and B are running).

Once the machine fails, i.e., when component A or B fails, the machine is repaired by replacing all the failed components with brand new ones. This repair process happens instantaneously and does not take any time.

(a) Assuming that the machine starts with brand new components at time 0, simulate the operation of the machine for 2 days. Use the uniform(0,1) random variates given in the table below (you may not need to use all):

Component A (failure)	0.91	0.76	0.11	0.72	0.67	0.57	0.83	0.29
Component B (failure)	0.17	0.12	0.64	0.04	0.83	0.91	0.22	0.30
Component C (failure)	0.87	0.09	0.96	0.95	0.54	0.94	0.76	0.76
Component A (price)	0.45	0.05	0.30	0.10	0.64	0.46	0.12	0.19

(b) Using your simulation estimate:

- The average cost per day
- The average proportion of time the component C is in the failed state when the machine is running.

Question2

(a) Consider the linear congruential generator (LCG) given below:

$$Z_{n+1} = 2737Z_n + 1251 \quad \text{mod}(4096)$$

Starting from $Z_0 = 1642$ and using Z_1 to generate your first uniform(0,1) random variable, generate 5 random variates and calculate the Kolmogorov-Smirnov test statistic using $(\sqrt{n}+0.12+\frac{0.11}{\sqrt{n}})D_n$, where D_n is the supremum distance between the empirical cumulative distribution and the uniform(0,1) cumulative distribution.

(b) In addition to the 5 random variates generated in (a), generate one more random variate using the LCG in (a) to have a total of 6 random variates. Form 3 two-dimensional vectors using these random variates. Then, dividing each axis into two, i.e., $k = 2$, find the test statistic for the serial test.

(c) Outline an inversion method with the following probability density function (pdf):

$$f(x) = \begin{cases} \frac{1}{6} & 0 \le x < 3 \\ \frac{1}{18} & 3 \le x < 4 \\ \frac{2}{9} & 4 \le x < 6 \\ 0 & \text{otherwise.} \end{cases}$$

Using the uniform(0,1) random variates generated in (a) generate 3 random variates with the pdf $f(x)$.

(d) Outline a composition method to generate random variates with the pdf given in part (c). Using the uniform(0,1) random variates generated in (a), generate 1 random variate with the pdf $f(x)$.

(e) A random variable is said to follow a truncated Normal(μ, σ^2) between a and b if the shape of its pdf is similar in shape to Normal$(\mu,\ \sigma^2)$ between a and b and 0 elsewhere. More precisely, a truncated normal random variable with mean μ and and variance σ^2 has the pdf

$$f(x) = \begin{cases} \dfrac{1}{C\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{\sigma^2}} & a < x < b \\ 0 & \text{otherwise,} \end{cases}$$

where $C = \displaystyle\int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{\sigma^2}} dx$. Using the fact that we can generate a standard normal random variate using the formula

$$Z = \sqrt{-2 \ln U_1} \cos 2\pi U_2$$

where U_1 and U_2 are uniform(0,1) random variables, outline a method to generate truncated Normal(μ, σ^2) between a and b.