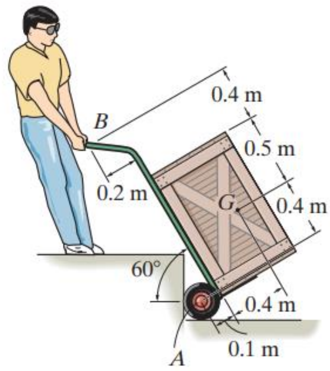
ESCUELA SUPERIOR POLITECNICA DEL LITORAL
FACULTAD DE INGENIERIA MECANICA Y CIENCIAS DE LA PRODUCCION
TERCERA EVALUACION DE MECANICA VECTORIAL SEGUNDO TERMINO 2024

Paralelo: Fecha: 7 de febrero del 2025 Profesor:

Nombre………………………………………CI.:…………………………Firma:……………………

PRIMER TEMA: Equilibrio de Solidos Rígidos (20%)

Un estibador usa una carreta para mover una caja hacia arriba de la grada. Si la carreta y su contenido tienen una masa de 50 kg con centro de gravedad en G, determine la reaccion normal en ambas ruedas y la minima fuerza requerida en el manubrio B para subir la caja.



Equations of Equilibriums. $\mathbf{P}_y$ can be determined directly by writing the force equation of equilibrium along y axis by referring to the *FBD* of the hand truck shown in Fig. *a*.

$$+\uparrow \Sigma F_y = 0; \qquad P_y - 50(9.81) = 0 \qquad P_y = 490.5\text{ N}$$

Using this result to write the moment equation of equilibrium about point A,

$$\circlearrowleft + \Sigma M_A = 0; \qquad P_x \sin 60^\circ(1.3) - P_x \cos 60^\circ(0.1) - 490.5 \cos 30^\circ(0.1)$$
$$-490.5 \sin 30^\circ(1.3) - 50(9.81) \sin 60^\circ(0.5)$$
$$+50(9.81) \cos 60^\circ(0.4) = 0$$
$$P_x = 442.07\text{ N}$$

Thus, the magnitude of minimum force $\boldsymbol{P}$, Fig. *b*, is

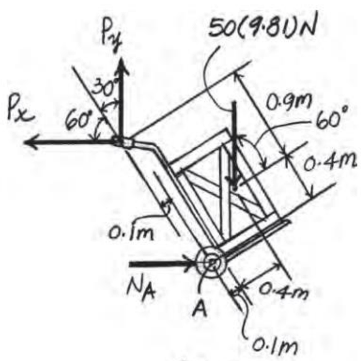
$$P = \sqrt{P_x^2 + P_y^2} = \sqrt{442.07^2 + 490.5^2} = 660.32\text{ N} = 660\text{ N} \qquad \textbf{Ans.}$$

and the angle is

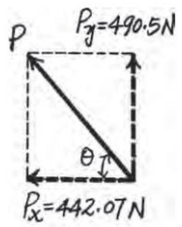
$$\theta = \tan^{-1}\left(\frac{490.5}{442.07}\right) = 47.97^\circ = 48.0^\circ \qquad \textbf{Ans.}$$

Write the force equation of equilibrium along x axis,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_A - 442.07 = 0 \qquad N_A = 442.07\text{ N} = 442\text{ N} \qquad \textbf{Ans.}$$



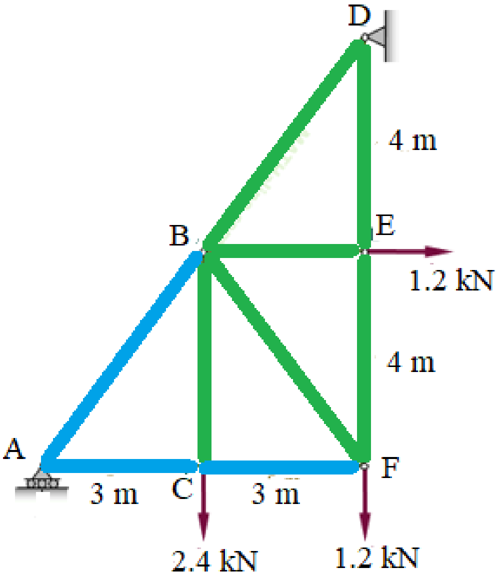
(a)



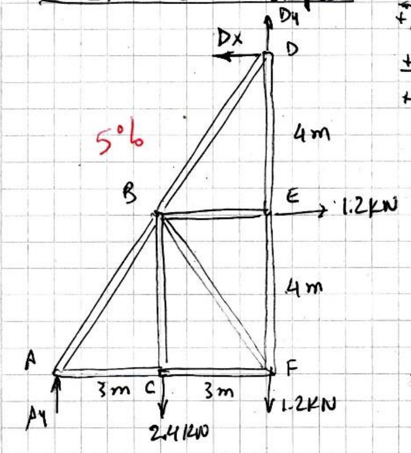
(b)

SEGUNDO TEMA: Estructuras (20%)

Por el método de los nodos determine la fuerza presente en cada elemento de la estructura y establezca si estas se encuentran en Tensión o en Compresión.



DCL: armadura completa



5%

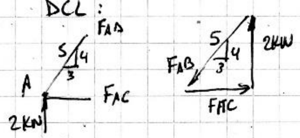
$\circlearrowright \Sigma M_D = 0: \quad 4(1.2) + 3(2.4) - 6A_y = 0 \Rightarrow A_y = 2\,kN \uparrow$ 2%

$\xrightarrow{+} \Sigma F_x = 0: \quad -D_x + 1.2\,kN = 0 \Rightarrow D_x = 1.2\,kN \leftarrow$ 2%

$+\uparrow \Sigma F_y = 0 \quad D_y - 2.4 - 1.2 + 2 = 0 \Rightarrow D_y = 1.6\,kN \uparrow$ 2%

Nodo A

DCL:



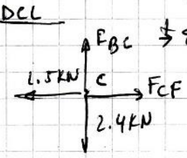
$\frac{2}{4} = \frac{F_{AC}}{3} = \frac{F_{AB}}{5}$

$\Rightarrow F_{AC} = 1.5\,kN$ T 1%

$\Rightarrow F_{AB} = 2.5\,kN$ C 1%

Nodo C

DCL



$+\uparrow \Sigma F_y = 0 \quad F_{BC} - 2.4\,kN = 0$

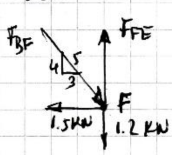
$F_{BC} = 2.4\,kN$ T 1%

$\xrightarrow{+} \Sigma F_x = 0 \quad F_{CF} - 1.5\,kN = 0$

$F_{CF} = 1.5\,kN$ T 1%

Nodo F

DCL

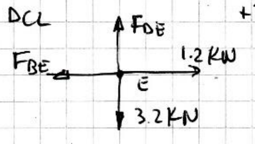


$\xrightarrow{+} \Sigma F_x = 0 \quad \frac{3}{5} F_{BF} - 1.5\,kN = 0 \quad F_{BF} = 2.5\,kN$ C 1%

$+\uparrow \Sigma F_y = 0 \quad F_{FE} - 1.2\,kN - \frac{4}{5}(2.5\,kN) = 0 \Rightarrow F_{FE} = 3.2\,kN$ T 1%

Nodo E:

DCL

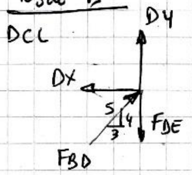


$\xrightarrow{+} \Sigma F_x = 0 \quad 1.2\,kN - F_{BE} = 0 \Rightarrow F_{BE} = 1.2\,kN$ T 1%

$+\uparrow \Sigma F_y = 0 \quad F_{DE} - 3.2\,kN = 0 \Rightarrow F_{DE} = 3.2\,kN$ T 1%

Nodo D

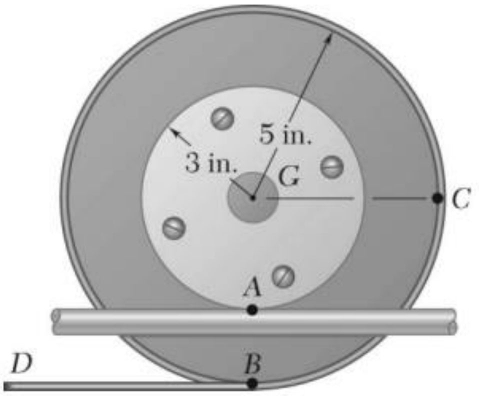
DCL



$\xrightarrow{+} \Sigma F_x = 0 \quad \frac{3}{5} F_{BD} - 1.2\,kN = 0 \Rightarrow F_{BD} = 2\,kN$ C 1%

TERCER TEMA: Cinemática (30%)

Un disco de 3 in de radio esta rigidamente unido a un tambor de 5 in de radio como se muestra. Uno de los tambores rueda sin deslizar sobre la superficie mostrada, y una cuerda esta enrollada al otro tambor. Sabiendo que en el instante mostrado el extremo D de la cuerda tiene una velocidad de 8 in/s y una aceleracion de 30 in/s, ambas en direccion hacia la izquierda, determine las aceleraciones de los puntos A, B y C de los tambores.



Velocity analysis. $\mathbf{v}_D = \mathbf{v}_B = 8$ in./s

Instantaneous center is at Point *A*. $v_B = (AB)\omega, \quad 8 = (5-3)\omega$

$\omega = 4$ rad/s ↻

Acceleration analysis. $\mathbf{a}_A = [a_A \uparrow]$ for no slipping. $\alpha = \alpha$ ↻

$\mathbf{a}_B = [30 \text{ in./s}^2 \leftarrow] + [(a_B)_n \uparrow]$

$\mathbf{a}_G = [a_G \rightarrow]$

$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_t + (\mathbf{a}_{A/B})_n$

$[a_A \uparrow] = [30 \leftarrow] + [(a_B)_n \uparrow] + [(5-3)\alpha \rightarrow] + [(5-3)]\omega^2 \downarrow]$

Components $\xrightarrow{+}$: $0 = -30 + 2\alpha \qquad \alpha = 15 \text{ rad/s}^2$ ↻

$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$

$a_A \uparrow = [a_G \rightarrow] + [3\alpha \leftarrow] + [3\omega^2 \uparrow]$

Components $\xrightarrow{+}$: $0 = a_G - 3\alpha \qquad a_G = 3\alpha = 45 \text{ in./s}^2$

$+\uparrow$: $a_A = 3\omega^2 = (3)(4)^2 = 48 \text{ in./s}^2 \qquad \mathbf{a}_A = 48.0 \text{ in./s}^2 \uparrow$

$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$

$= [45 \rightarrow] + [5\alpha \leftarrow] + [5\omega^2 \uparrow]$

$= [45 \rightarrow] + [75 \leftarrow] + [80 \uparrow]$

$= [30 \text{ in./s}^2 \leftarrow] + [80 \text{ in./s}^2 \uparrow]$

$\mathbf{a}_B = 85.4 \text{ in./s}^2 \measuredangle 69.4°$

$$\mathbf{a}_C = \mathbf{a}_G + (\mathbf{a}_{C/G})_t + (\mathbf{a}_{C/G})_n$$

$$= [45 \longrightarrow] + [5\alpha \downarrow] + [5\omega^2 \longleftarrow]$$

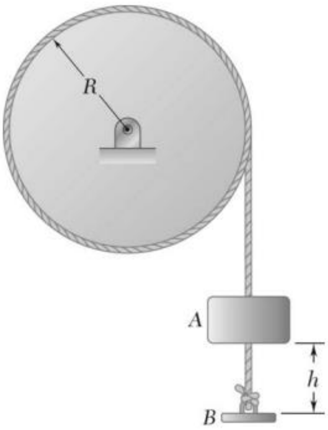
$$= [45 \longrightarrow] + [75 \downarrow] + [80 \longleftarrow]$$

$$= [35 \text{ in./s}^2 \longleftarrow] + [75 \text{ in./s}^2 \downarrow] \qquad \mathbf{a}_C = 82.8 \text{ in./s}^2 \swarrow 65.0^\circ$$

CUARTO TEMA: Impulso y cantidad de movimiento (30%)

Un plato B de masa despreciable es agarrado a la cuerda que esta enrollada alrededor del disco uniforme de 8 lb y de radio R= 9 in. Un collarín A de 3 lb es liberado desde el reposo y cae una distancia h = 15 in antes de impactar el plato B.

Si el coeficiente de restitución entre el collarín y el plato B es de 0.8 encuentre: a) la velocidad final del collarín, b) a velocidad angular final del disco.



$$W_D = 8 \text{ lb}$$

$$m_D = \frac{W_D}{g} = \frac{8}{32.2} = 0.2484 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$R = 9 \text{ in.} = 0.75 \text{ ft}$$

$$I_D = \frac{1}{2} m_D R^2 = 0.06988 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$W_A = 3 \text{ lb}$$

$$m_A = \frac{W_A}{g} = \frac{3}{32.2} = 0.09317 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$h = 15 \text{ in.} = 1.25 \text{ ft}$$

Collar A falls through distance h. Use conservation of energy.

$$T_1 = 0$$

$$V_1 = W_A h$$

$$T_2 = \frac{1}{2} m_A v_A^2$$

$$V_2 = 0$$

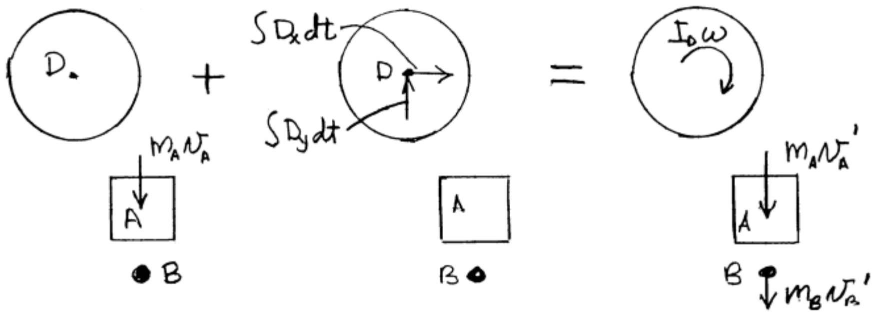
$$T_1 + V_1 = T_2 + V_2: \quad 0 + W_A h = \frac{1}{2} m_A v_A^2 + 0$$

$$v_A^2 = \frac{2m_A h}{W_A} = 2gh$$
$$= (2)(32.2)(1.25)$$
$$= 80.5 \text{ ft}^2/\text{s}^2$$
$$\mathbf{v}_A = 8.972 \text{ ft/s} \downarrow$$

Impact. Neglect the mass of plate B. Neglect the effect of weight over the duration of the impact.

Kinematics. $\quad \boldsymbol{\omega}' = \omega \circlearrowright \quad \mathbf{v}_B' = R\omega \downarrow = 0.75\omega' \downarrow$

Conservation of momentum.



$+\circlearrowright$ Moments about D: $\quad m_A v_A R + 0 = m_A v_A' R + I_D \omega' + m_B v_B' R$

$$(0.09317)(8.972)(0.75) = (0.09317)(0.75)v_A' + 0.06988\,\omega' \qquad (1)$$

Coefficient of restitution. $\quad v_B' - v_A' = e(v_A - v_B)$

$$0.75\omega' - v_A' = 0.8(8.972 - 0) \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously

(*a*) <u>Velocity of *A*</u>. $\quad v_A' = -0.25648$ ft/s $\quad \mathbf{v}_A' = 0.256 \text{ ft/s} \uparrow$ ◀

(*b*) <u>Angular velocity</u>. $\quad \omega' = 9.228$ rad/s $\quad \boldsymbol{\omega}' = 9.23 \text{ rad/s} \circlearrowright$ ◀